Engineering Mechanics :— It is science that describes and predicts the effect of force on object either at rest/motion.

Engg. Mechanics

- Mechanics of Solid
  - Deformable Bodies
  - Rigid Bodies
    - Statics
    - Dynamics
      - Kinematics
      - Kinetics
- Mechanics of Fluid

System of Forces :—

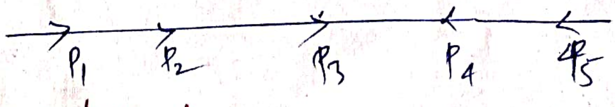
When several forces of different magnitude and direction act upon a body, they constitute a force system.

Considering the plane in which force applied and depending upon the position of line of action, forces may classified as—

System of forces

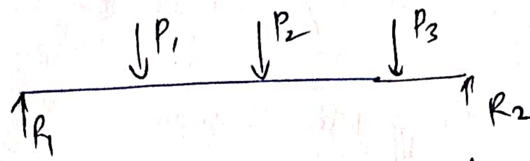
- Coplanar Forces
  - Concurrent
  - Parallel
    - Like
    - Unlike
  - Non-concurrent Non parallel
- Non-coplanar or Spatial forces
  - Concurrent
  - Parallel
  - Non concurrent Non parallel

• Collinear Forces :— The line of action of all forces lie along the same straight line and in the same plane



Exp → Force on a rope in a tug of war.

Coplanar parallel forces:— The line of action of all forces are parallel to each other and lie in a single plane.

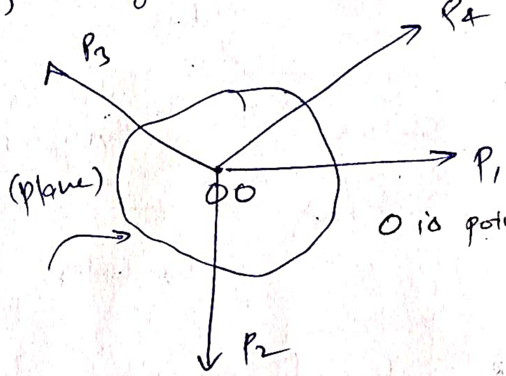


Exp→ System of vertical load (including reactions) acting on a beam.

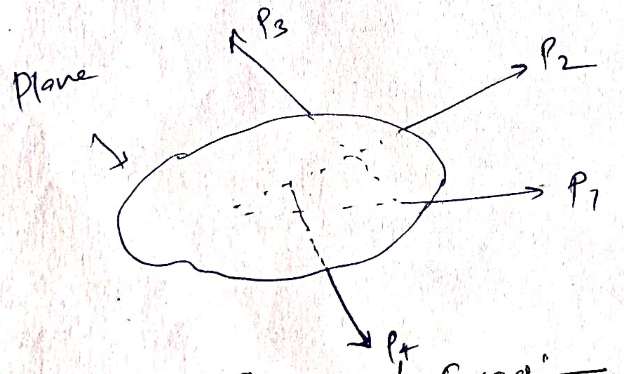
Coplanar concurrent forces:— All forces lie in the same plane, have different directions but their lines of action act at one point (pass through a single point).

The point where the lines of action of the forces meet is known as the point of concurrency of the force system.

Exp:— Forces on a rod resting against a wall.

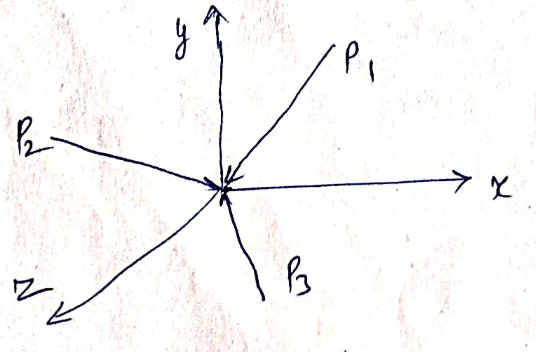


Coplanar non-concurrent forces:— All forces lie in the same plane but the line of action do not pass through a single point.



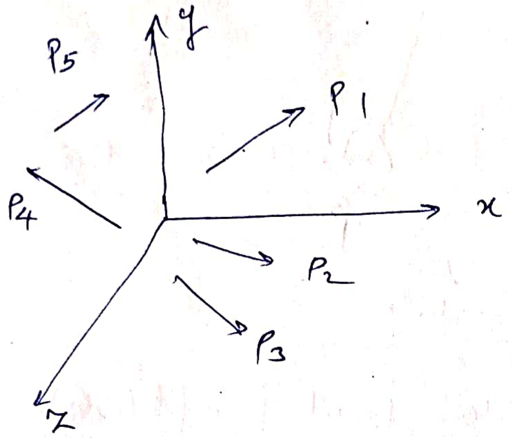
Exp:— Forces on a ladder resting against wall and a person standing on a rung which is not at its centre of gravity.

Non-coplanar concurrent forces:— All forces do not lie in same plane but their lines of action pass through a single point.



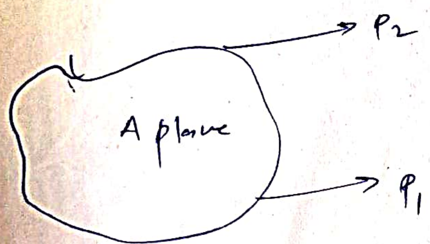
Exp:- Forces on a tripod carrying a camera.

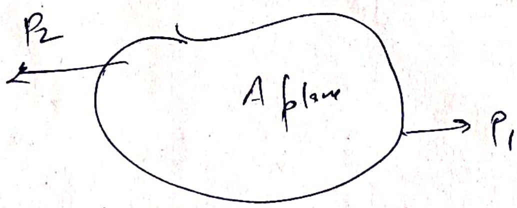
Non-coplanar & Non-concurrent forces:— All forces do not lie in the same plane, and their line of action do not meet at a single point.



Exp:— Forces acting on a moving bus.

Coplanar like and Unlike parallel force system:— The different forces in a coplanar parallel force system lie in the same plane and have their line of action parallel to each other.





(a)

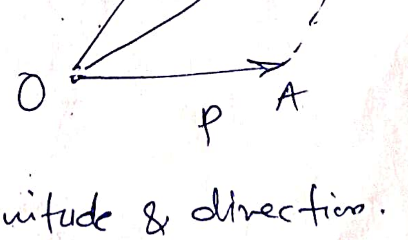
(b)

(i) The system is called the coplanar like parallel force system if the parallel forces point in the same direction.

(ii) The system is called the coplanar unlike parallel force system if the forces point in opposite directions.

Parallelogram Law of Forces:—

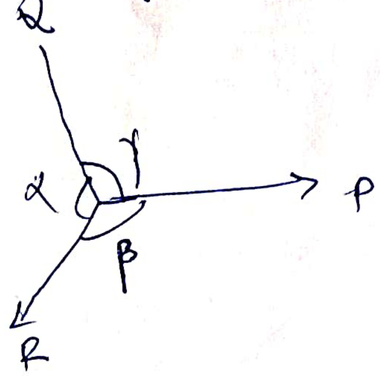
If two forces P & Q acting at a point are represented in magnitude and direction by the two adjacent sides of a parallelogram, then the diagonal passing through their point of intersection represents the resultant both in magnitude & direction.



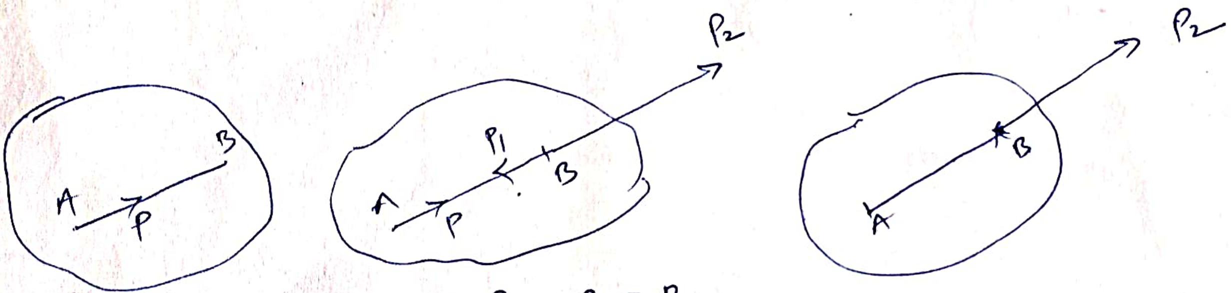
Lami's Theorem :-

"If three forces acting at a point are in equilibrium, then each forces is proportional to the sine of the angle b/w the other two forces."

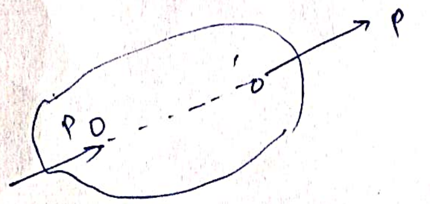
$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$



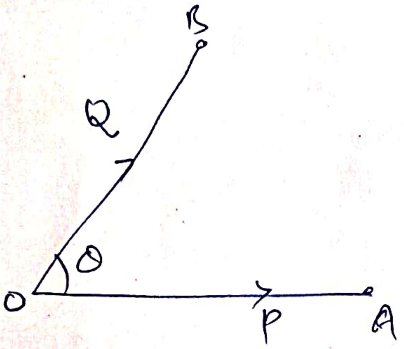
Principle of Transmissibility :- When the point of application of a force acting on a body is shifted to any other point on the line of action of the force without changing its direction, there occurs no change in the equilibrium state of the body.

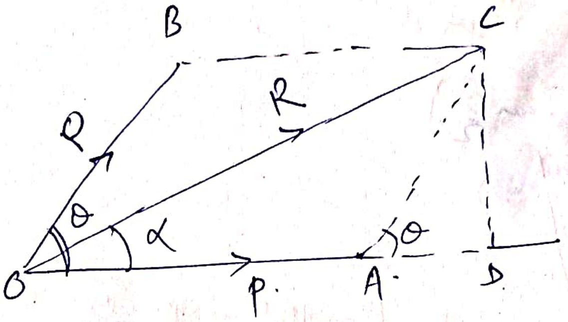


$P_1 = P_2 = P$



Parallelogram Law of Forces:- "If two forces, acting at a point be represented in magnitude and direction by the two adjacent side of a parallelogram, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through that point."





$OA = P$
$AD = AC\cos\theta$
$= Q\cos\theta$
$CD = AC\sin\theta$
$= Q\sin\theta$

$$R = OC = \sqrt{OD^2 + CD^2} = \sqrt{(OA+AD)^2 + CD^2}$$
$$= \sqrt{(P+Q\cos\theta)^2 + (Q\sin\theta)^2}$$
$$= \sqrt{P^2 + Q^2\cos^2\theta + 2PQ\cos\theta + Q^2\sin^2\theta}$$
$$\boxed{R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}}$$

The Inclination of resultant R to the direction of force P is

$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA+AD} = \frac{Q\sin\theta}{P+Q\cos\theta}.$$

$$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P+Q\cos\theta}\right]$$

Special Cases— ① when two forces are equal and θ is the angle b/w them

$$R = \sqrt{P^2+Q^2+2PQ\cos\theta} = \sqrt{P^2+P^2+2P\times P\times\cos\theta}$$
$$= \sqrt{2P^2(1+\cos\theta)}$$
$$= \sqrt{2P^2\times 2\cos^2\frac{\theta}{2}}$$
$$\boxed{R = 2P.\cos\frac{\theta}{2}}$$

$$\alpha = \tan^{-1}\left[\frac{Q\sin\theta}{P+Q\cos\theta}\right]$$

$$= \tan^{-1}\left[\frac{P\sin\theta}{P+P\cos\theta}\right] = \tan^{-1}\left(\frac{\sin\theta}{1+\cos\theta}\right)$$

$$= \tan^{-1}\left\{\frac{2\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}{2\cos^2\frac{\theta}{2}}\right\}$$

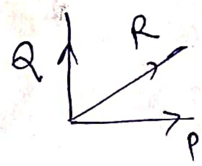
$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$\boxed{\alpha = \frac{\theta}{2}}$$

(II) When the two forces act at right angle ie. $\theta = 90°$ -

$$R = \sqrt{P^2+Q^2+2PQ\cos 90°} \qquad \cos 90 = 0$$

$$\boxed{R = \sqrt{P^2+Q^2}}$$

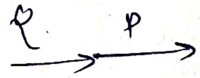


$$\alpha = \tan^{-1}\left\{\frac{Q\sin 90}{P+Q\cos 90}\right\}$$

$$\boxed{\alpha = \tan^{-1}\{Q/P\}}$$

(III) When the two forces act in same line & same sense ie. $\theta = 0°$.

$$R = \sqrt{P^2+Q^2+2PQ\cos 0}$$

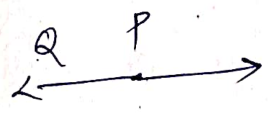


$$= \sqrt{P^2+Q^2+2PQ}$$

$$\boxed{R = P+Q}$$

(IV) When the two forces have same line of action but opposite sense ie $\theta = 180°$.

$$R = \sqrt{P^2+Q^2+2PQ\cdot\cos 180°}$$



$$= P^2+Q^2-2PQ$$

$$= (P-Q)$$

Two coplanar forces with magnitude 20 kN and 15 kN are oriented with angle of inclination equal to 60°. What would be their resultant in magnitude & direction if —

(a) both the forces act away from their point of intersection O,

(b) first force acts away from point O and second force acts towards the point.

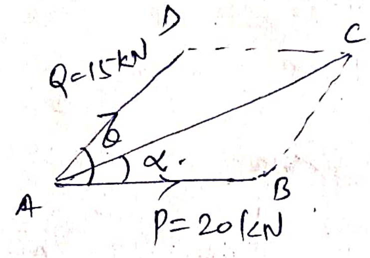
Soln:— Case (a) — $P = 20\ kN$, $Q = 15\ kN$ $\theta = 60°$

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{20^2 + 15^2 + 2 \times 20 \times 15 \cos 60}$$

$$= \sqrt{925}$$

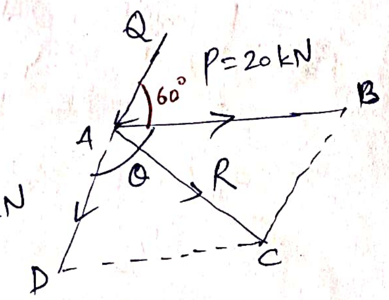
$$R = 30.41\ kN$$



$$\tan\alpha = \frac{Q\sin\theta}{P + Q\cos\theta} = \frac{15 \cdot \sin 60°}{20 + 15\cos 60} = \frac{15 \times 0.866}{20 + 15 \times 0.2}$$

$$\tan\alpha = 0.4723$$

$$\boxed{\alpha = 25.28° \text{ (clockwise)}}$$



Case (b):—

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$$

$$= \sqrt{20^2 + 15^2 + 2 \times 20 \times 15 \times \cos 120°}$$

$$= \sqrt{325}$$

$$\boxed{R = 18.03\ N}$$

$$\tan\alpha = \frac{Q \cdot \sin\theta}{P + Q \cdot \cos\theta} = \frac{15 \cdot \sin 120}{20 + 15\cos 120°}$$

$$= 1.0392$$

$$\boxed{\alpha = 46.1°}$$

Q① Resultant of two equal forces is equal to either of them. Determine the angle b/w the forces.

② The resultant of two forces $(P+Q)$ & $(P-Q)$ equal $\sqrt{3P^2+Q^2}$. Show that the forces are then inclined to each other at an angle of $60°$.

③ The resultant of two forces P & Q is R. If one of the forces is reversed in direction. The resultant is S. Then for the identity.

$$R^2+S^2 = 2(P^2+Q^2)$$

to hold good, show that the forces can have any angle of inclination b/w them.

Solⁿ:– (I) $R = 2P\cos\frac{\theta}{2}$,

Since the resultant equal either of the two equal forces –

$P = 2P\cos\frac{\theta}{2}$. $\cos\frac{\theta}{2} = \frac{1}{2}$ ⇒ $\frac{\theta}{2} = 60°$

$\theta = 120°$, or $\frac{2\pi}{3}$.

(II) $R^2 = P^2 + Q^2 + 2P \cdot Q \cdot \cos\theta$ $P = P+Q$, $Q = (P-Q)$

$$3P^2+Q^2 = (P+Q)^2 + (P-Q)^2 + 2(P+Q)(P-Q)\cdot\cos\theta$$
$$= P^2+Q^2+2PQ+P^2+Q^2-2PQ+2(P^2-Q^2)\cdot\cos\theta$$

$$3P^2+Q^2 = 2P^2+2Q^2+2(P^2-Q^2)\cdot\cos\theta$$
$$P^2-Q^2 = 2(P^2-Q^2)\cdot\cos\theta$$
$$\cos\theta = 1/2, \quad \theta = 60°, \ \frac{\pi}{3}$$

(III)

$$R^2 = P^2+Q^2+2PQ\cdot\cos\theta \quad -(I)$$
$$S^2 = P^2+(-Q)^2+2P\times(-Q)\cdot\cos\theta$$
$$= P^2+Q^2-2P\cdot Q\cdot\cos\theta \quad -(II)$$

– Adding eq (I) & II –

$$\boxed{R^2+S^2 = 2(P^2+Q^2)}$$

Q: The magnitude of two forces is such that when acting at right angles produce a resultant force of $\sqrt{20}$ and when acting at 60° produce a resultant equal to $\sqrt{28}$. Work out the magnitude of the two forces.

Soln:-

$$R^2 = P^2 + Q^2 + 2PQ.\cos\theta$$

$$\rightarrow 20 = P^2 + Q^2 + 2P.Q.\cos 90^\circ \qquad \because \cos 90^\circ = 0$$

$$P^2 + Q^2 = 20 \quad \text{— (I)}$$

$$28 = P^2 + Q^2 + 2P.Q.\cos 60^\circ \qquad \cos 60^\circ = 1/2$$

$$28 = P^2 + Q^2 + PQ \quad \text{— (II)}$$

$$PQ = 8, \qquad Q^2 = \frac{64}{P^2}$$

$$\Rightarrow 20 = P^2 + \frac{64}{P^2}$$

$$P^4 - 20P^2 + 64 = 0$$

$$(P^2 - 16)(P^2 - 4) = 0$$

$$\boxed{P = 2, \text{ or } 4 \text{ units}}$$

Q. The greatest & least resultant of two forces acting on a body are 35kN & 5kN respectively. Determine the magnitude of these forces. What would be the angle these forces if the magnitude of the resultant is stated to be 25kN?

Soln:- Resultant will be maximum when the forces are collinear & act in the same direction, i.e. $\theta = 0^\circ$, — $\qquad \cos 0^\circ = 1$

$$R^2 = P^2 + Q^2 + 2PQ.\cos 0^\circ$$

$$R = P + Q \Rightarrow \quad 35 = P + Q \quad \text{— (I)}$$

Resultant will be minm when the forces are collinear and act in the opposite direction i.e $\theta = 180^\circ$ —

$$R = \sqrt{P^2 + Q^2 + 2PQ\cos 180^\circ}$$

$$R = P - Q \Rightarrow \quad P - Q = 5 \quad \text{— (II)}$$

from eqn (1) & (2) $\longrightarrow$ $P = 20$ kN, $Q = 15$ kN.

(b) Let θ be the angle b/w the force $\quad P = 20\text{kN}, \; Q = 15\text{kN} \quad R = 25$.

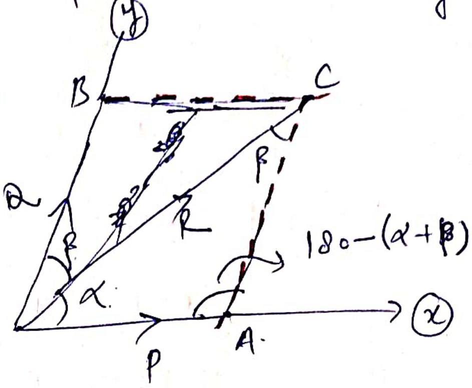
$$R^2 = P^2 + Q^2 + 2PQ.\cos\theta$$

$$25^2 = 20^2 + 15^2 + 2 \times 20 \times 15 \times \cos\theta$$

$$625 = 400 + 225 + 600\cos\theta \Rightarrow \quad \cos\theta = 0$$

$$\boxed{\theta = 90^\circ}$$

Resolution of Forces :— Finding the components of a given force in two given directions is called resolution. These component forces will have the same effect on the body as given single force.



$OA = P, \quad OB = Q$

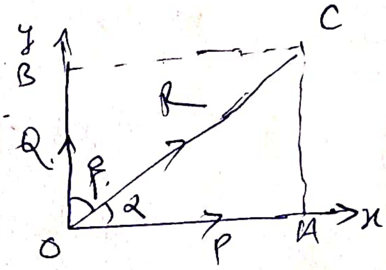
$\angle OCA = \angle BOC$ (alternate angles)

$= \beta$

$\angle OAC = 180^\circ - (\alpha + \beta)$

Applying sine rule to ΔOAC we get —

$$\frac{OA}{\sin\beta} = \frac{AC}{\sin\alpha} = \frac{OC}{\sin\{180 - (\alpha+\beta)\}}$$



AC is parallel to $OB = Q$.

$$\frac{P}{\sin\beta} = \frac{Q}{\sin\alpha} = \frac{R}{\sin(\alpha+\beta)}$$

$$\boxed{P = \frac{R \cdot \sin\beta}{\sin(\alpha+\beta)} \quad \& \quad Q = R \cdot \frac{\sin\alpha}{\sin(\alpha+\beta)}} \quad \text{— (I)}$$

When the force R is to resolved along $\perp^r$ direction then — OX & OY are right angle & OACB become a rectangle

$\alpha + \beta = 90 \qquad \beta = 90 - \alpha$

$$P = R\frac{\sin\beta}{\sin(\alpha+\beta)} = R \cdot \frac{\sin(90-\alpha)}{\sin 90}$$

$$\boxed{P = R\cos\alpha} \quad \text{— (II)}$$

~~The projec~~ $Q = R\dfrac{\sin\alpha}{\sin(\alpha+\beta)} = R \cdot \dfrac{\sin\alpha}{\sin 90^\circ}$

$$\boxed{Q = R\sin\alpha}$$

Q. Determine the magnitude and direction of the resultant of the following set of forces acting on a body —

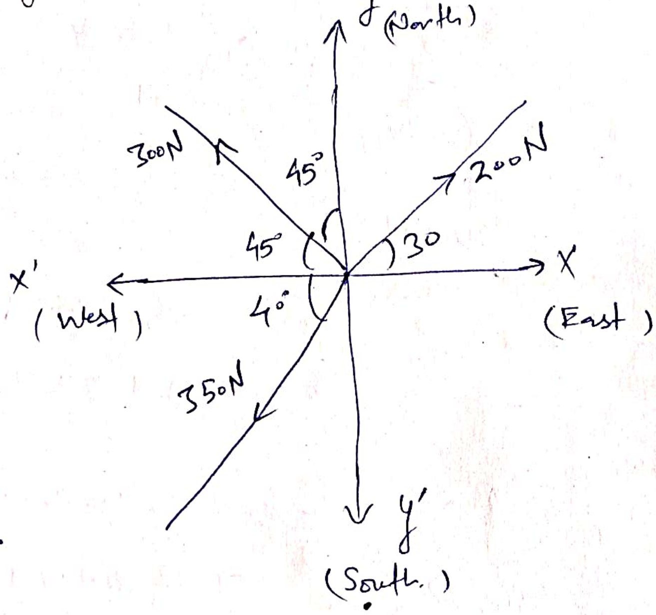
① 200 N inclined 30° with east towards North,

② 250 N towards the north,

③ 300 N towards North-west &

④ 350 N inclined at 40° with west towards south.

What will be the equilibrant of the given force system.



Sol^n:—

Resolving all the forces along — x direction —

$$\Sigma F_x = 200\cos 30° + 250\cos 90° + 300\cos 135° + 350\cos 220°$$

$$= -307\ N$$

$$\Sigma F_y = 200\sin 30° + 250\sin 90° + 300\sin 135° + 350\sin 220°$$

$$= 337.4\ N$$

$$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

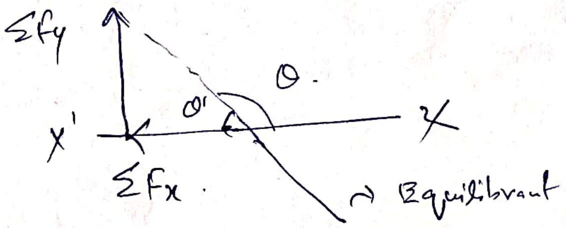
$$R = 456\ N$$

$$\theta' = \tan^{-1}\left(\frac{\Sigma F_y}{\Sigma F_x}\right)$$

{Since ΣF_x is −ve & ΣF_y is +ve} Resultant lies in IInd Quadrant

$$= \tan^{-1}\left(\frac{337.4}{307}\right)$$

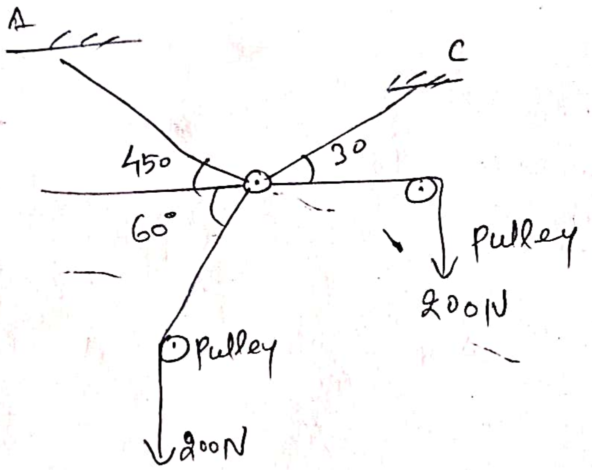
$$= 47.7°$$



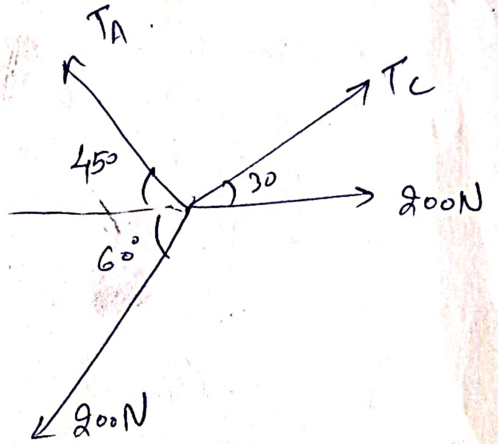
$$\theta = 180 - 47.7$$

$$= 132.3°$$

Q. Calculate the tensile force in the cable AB & BC as shown in fig. Presume the Pulleys to be frictionless —



FoB D'



$\Sigma F_x = 0 \Rightarrow 0 = -T_A \cos 45 + T_C \cos 30 + 200 - 200 \cos 60^\circ$

$\Rightarrow -T_A + 1.22 T_C + 141 = 0$ —— (I)

$\Sigma F_y = 0 \qquad 0 = T_A \sin 45^\circ + T_C \sin 30^\circ - 200 \sin 60^\circ$

$\Rightarrow \qquad T_A + 0.707 T_C - 244 = 0$ —— (II)

By solving (1) & (2) —

$\boxed{T_c = 53.45\,N}$

$\boxed{T_A = 206.21\,N}$

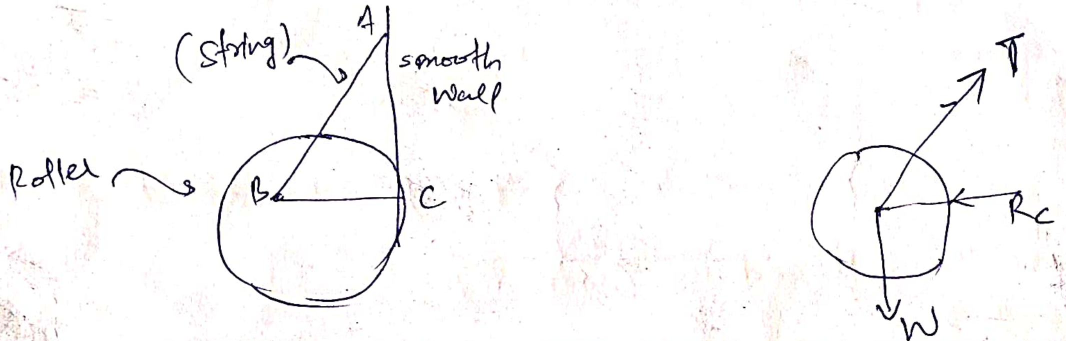
Free Body Diagram:—

(1) A sphere resting on a frictionless plane surface:—

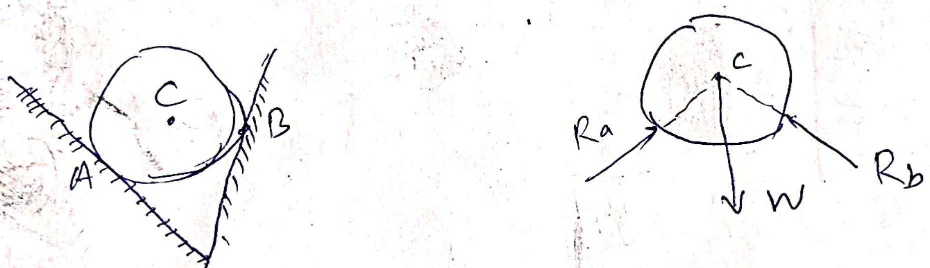
(a) Force W equal to the weight of the sphere, acts downward through the centroid of the sphere.

(b) Reaction R at the point of contact with the surface. Acts upwards normal to the surface as its is frictionless.

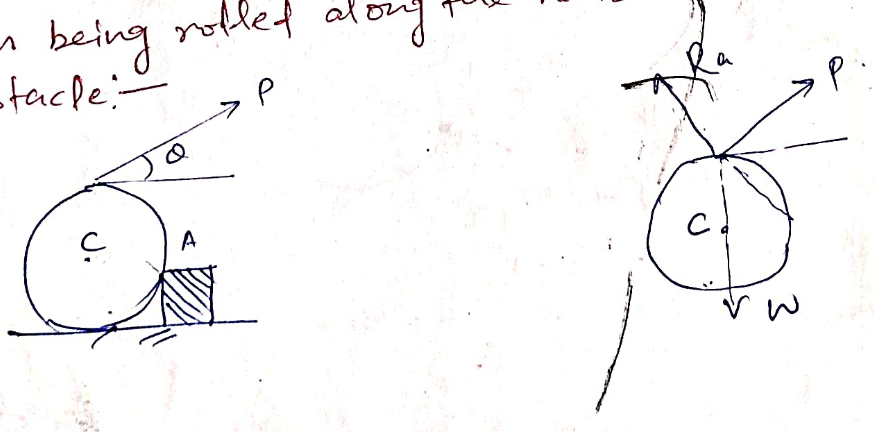
(2) A circular Roller of weight W hangs by a string and rest against a smooth vertical wall.



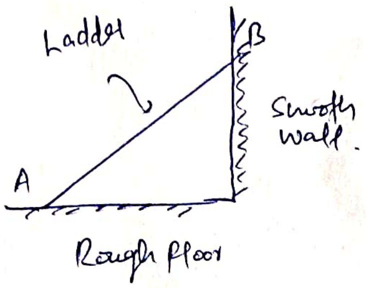
(3) A sphere resting in a V-shaped groove:—

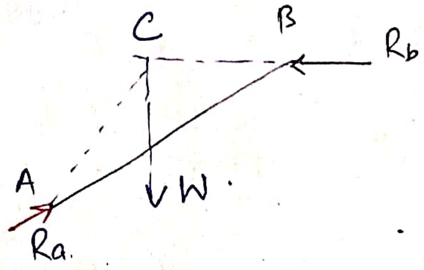


(4) A drum being rolled along the horizontal comes across a stepped obstacle:—

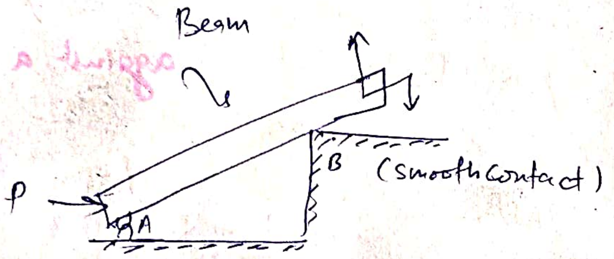


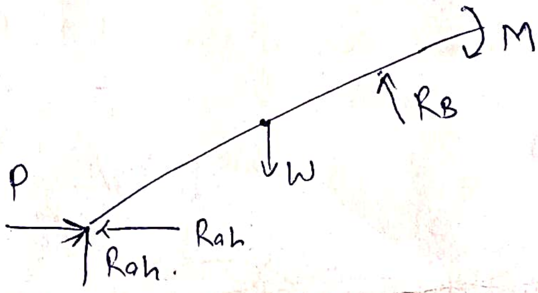
⟨5⟩ A uniform ladder of weight W leans against a smooth wall and rest on a rough floor.



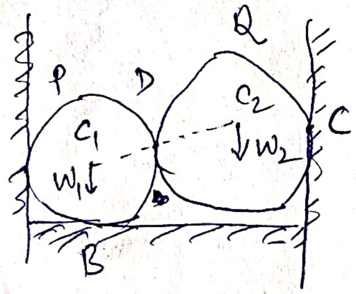


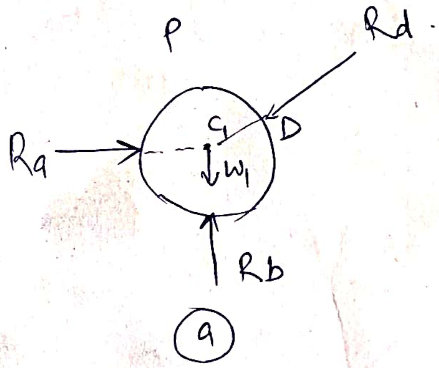
⟨6⟩ A Beam loaded and supported:—

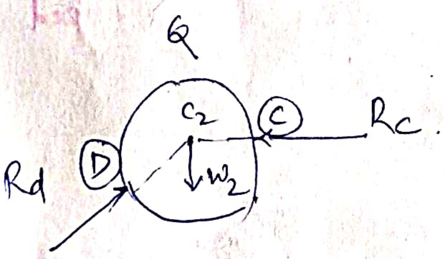


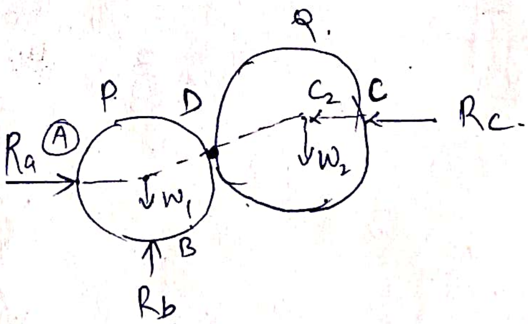


⟨7⟩ Two sphere P & Q placed in a vessel

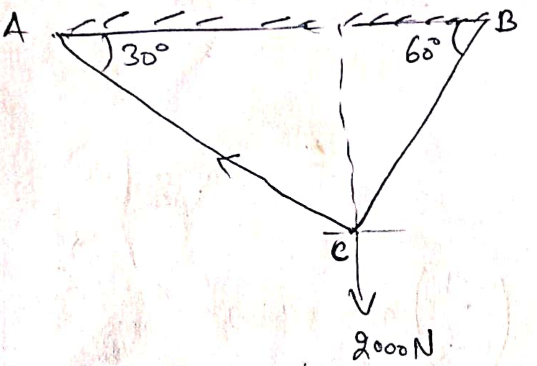


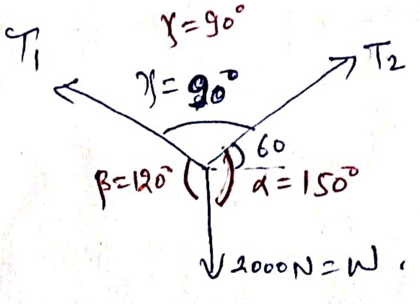






Q. A weight of 2000N is supported by two chain AC & BC as shown in Fig. Determine the Tension in each chain.



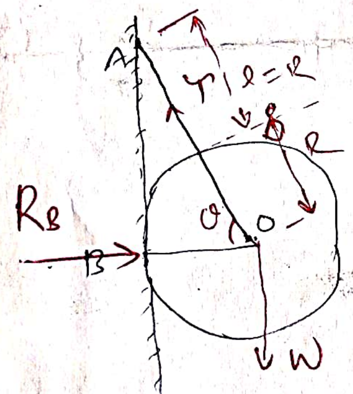


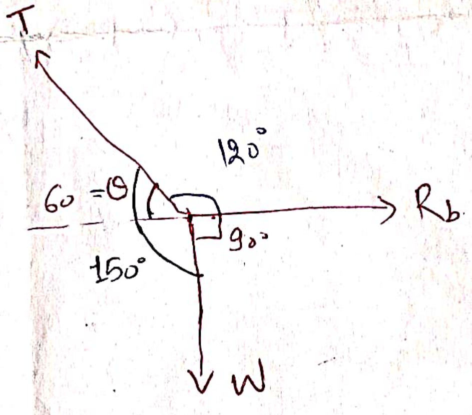
$$\frac{T_1}{\sin\alpha} = \frac{T_2}{\sin\beta} = \frac{W}{\sin\gamma} \Rightarrow \frac{T_1}{\sin 150} = \frac{T_2}{\sin 120°} = \frac{2000}{\sin 90°}$$

$T_1 = 1000\ N$

$T_2 = 1732\ N$

Q. A smooth sphere of radius 15 cm and weight 2N is supported in contact with a smooth vertical wall by a string whose length equals to the radius of a sphere. The string joins a point on the wall and a point on the surface of sphere. Workout inclination and find tension in the string and reaction of the wall.





In Δ AOB—

$$\cos\theta = \frac{OB}{OA} = \frac{R}{2R} = \frac{1}{2}$$

$\Rightarrow \theta = 60°$

$$\frac{T}{\sin 90} = \frac{W}{\sin 120°} = \frac{R_b}{\sin 150°}$$

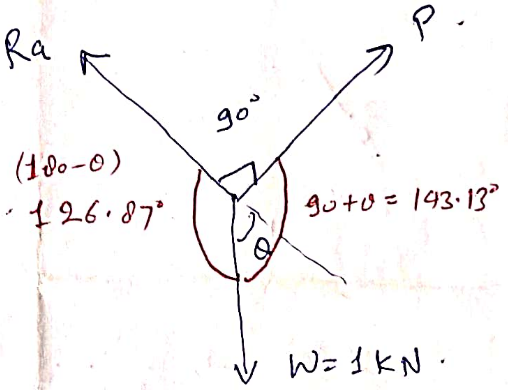
$$T = W \times \frac{\sin 90°}{\sin 120°} \Rightarrow T = 2.31\ N$$

$$R_b = W \times \frac{\sin 150°}{\sin 120°} = \cancel{2 \times .95} \Rightarrow R_b = 1.15\ N$$

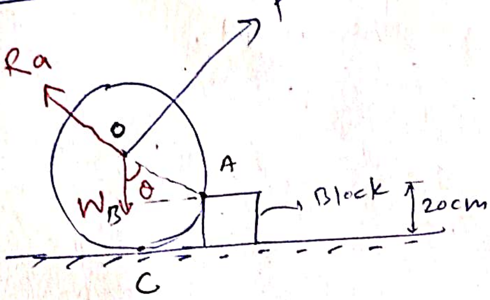
Q. A uniform wheel of 50cm diameter and 1 KN weight rests against a rigid rectangular block of thickness 20 cm. considering all surfaces smooth Determine —

(i) least pull to be applied through the centre of wheel just turn it over the corner of the block.

(ii) Reaction of Block —



Soln:—



ΔAOB

$$\cos\theta = \frac{OB}{OA} = \frac{50-20}{50}$$

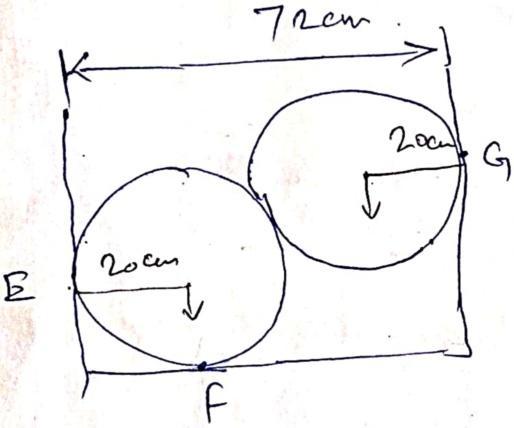
$\theta = 53.13°$

$$\frac{W}{\sin 90°} = \frac{P}{\sin 126.87} = \frac{Ra}{143.13°}$$

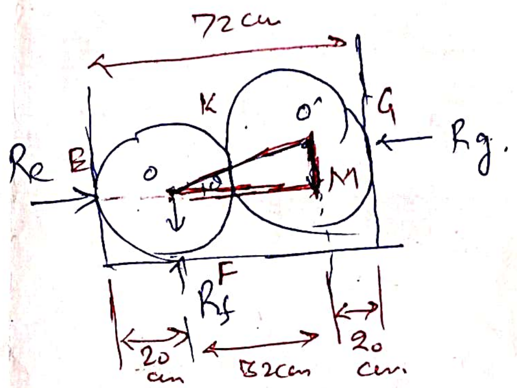
$P = W \times \frac{\sin 126.87'}{\sin 90°} \Rightarrow P = 0.8\text{ KN}$

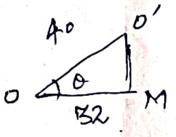
$Ra = W \times \frac{\sin 143.13°}{\sin 90°} \Rightarrow Ra = 0.6\text{ KN}$

Q. Two smooth sphere, each of radius 20 cm & weight 200N, rest in a horizontal channel having vertical walls, the distance b/w them is 72 cm as shown in fig. Find the pressures at contact point E, F & G.

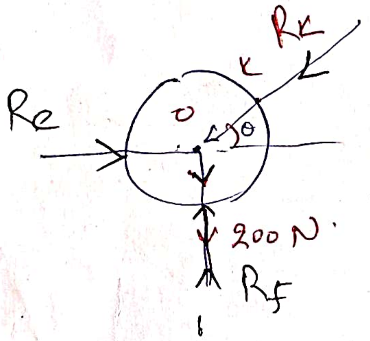


Sol^n:-

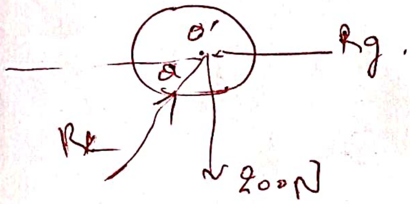




$\cos\theta = \frac{OM}{OO'} \Rightarrow \boxed{\theta = 36.87^\circ}$



Lower sphere



Upper sphere

$\Sigma F_x = 0$,

$R_k \cos\theta - R_g = 0$

$\Sigma F_y = 0$, $R_k \sin\theta - 200 = 0$

$R_k = \frac{200}{\sin\theta} \Rightarrow \boxed{R_k = 333.33N}$

$R_g = R_k \cos\theta$

$\boxed{R_g = 266.66N}$

$\Sigma F_x = 0$

$R_k \cos\theta - R_e = 0$

$\Sigma F_y = 0$,

$R_k \sin\theta + 200 - R_f = 0$

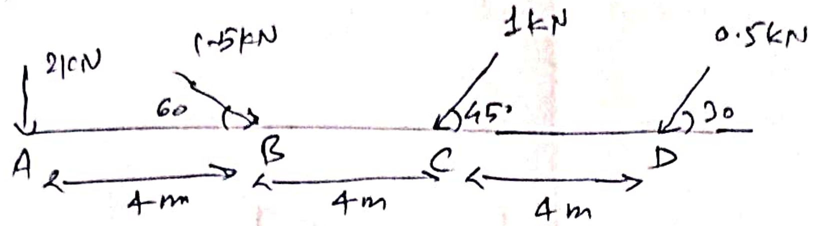
$R_e = R_k \cos\theta = 333.33 \times \cos 36.87$

$\boxed{R_e = 266.66 N}$

$R_f = R_k \sin\theta + 200$

$\boxed{R_f = 400 N}$

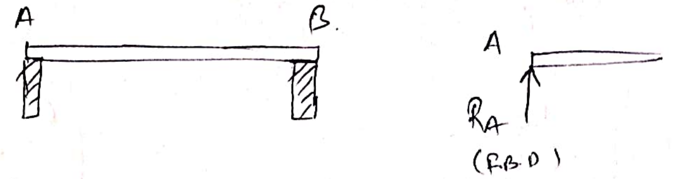
Q. A horizontal beam AD of length 12 m is acted upon by a set of forces shown in fig.



Determine the magnitude, direction & position of the resultant.

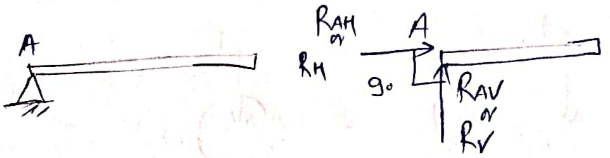
Types of support :-

① Simple Support :-

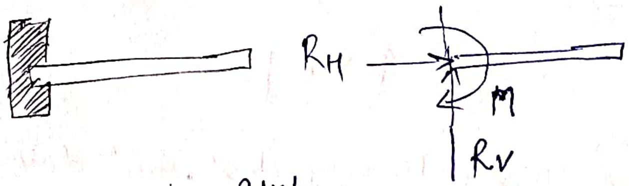


② Roller Support :-

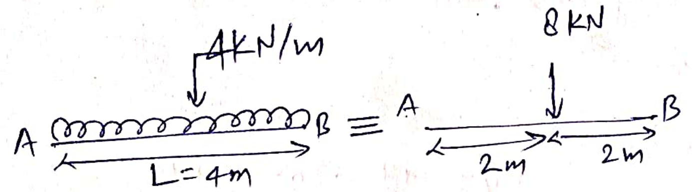
③ Hinge Support :-



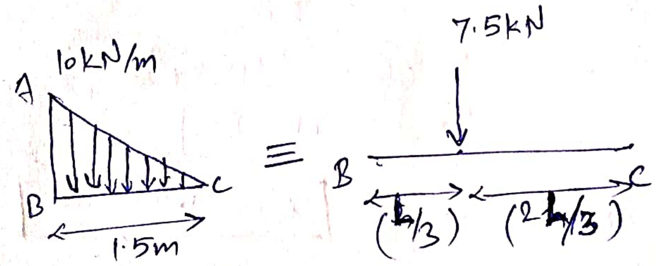
④ Fixed Support :-



U.D.L ⇒



U.V.L ⇒



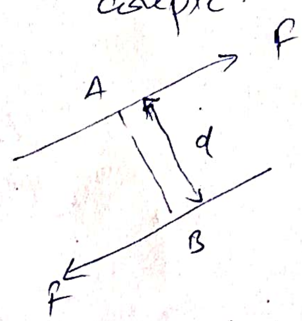
Total load $= \frac{1}{2} \times 1.5 \times 10$ kN

$= 7.5$

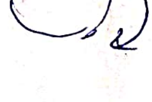
Varignon's Theorem: Law of Moments :-

• Moment of a resultant of two forces, about a point lying in the plane of force, is equal to the algebraic sum of the moments of these two forces about the same point.

Couple :- Two parallel forces equal in magnitude but opposite in direction and separated by a finite distance are said to form a couple.



Moment of couple $M = F \times d$



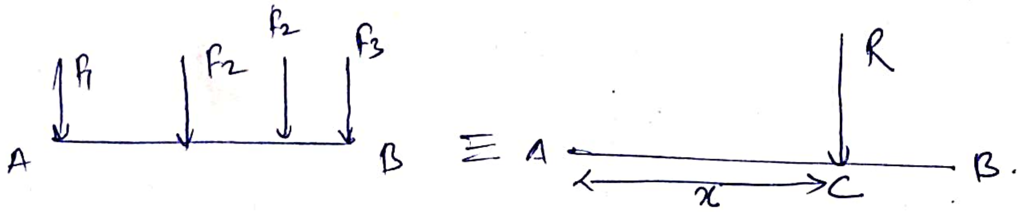
Exp. of a couple :-

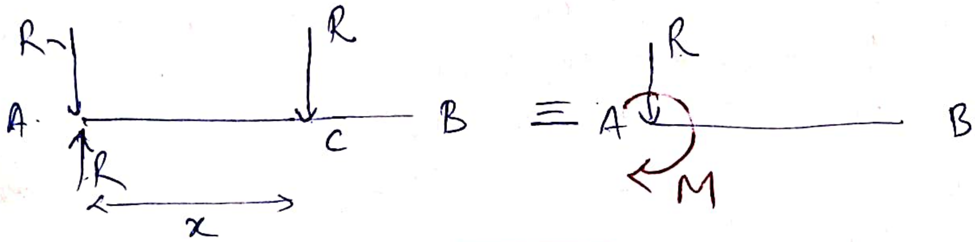
① opening or closing a water tap

② forces applied to the handle of a screw.

③ locking & unlocking of lock with a key.

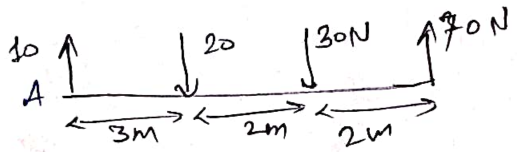
Resolution of force system into a force and a couple :-





$$\boxed{M_A = R \times x}$$

Q. Locate the resultant with magnitude and direction for a parallel force system shown in figure –



$\Sigma F_x = 0$

$\Sigma F_y = +10 - 20 - 30 + 70$
$= 30$

$R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\boxed{R = 30\text{N} \uparrow}$

For location –

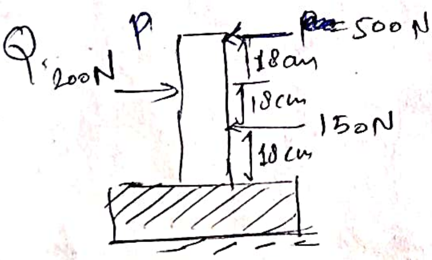
Applying Varignon's Theorem.

$\Sigma M_A = R \times x \Rightarrow 280 = 30 \times x \Rightarrow \boxed{x = 9.333\text{m}}$

$\circlearrowleft + \; \Sigma M_A = 20 \times 3 + 30 \times 5 - 70 \times 7 =$
$= -280 \text{ N-m}$

$\Sigma M_A = 280 \text{ N-m (A.C.W)}$

Q.



$\xrightarrow{+} \Sigma F_x = 200 - 500 - 150$
$= -450$

$+\Sigma F_y = 0$

$R = \sqrt{\Sigma F_x^2 + \Sigma F_y^2}$

$\boxed{R = 450 \text{ N}}$

$\circlearrowleft + \; \Sigma M_P = -\cancel{3}00 \times 18 + 150 \times 36$
$= 1800 \text{ N-cm}$

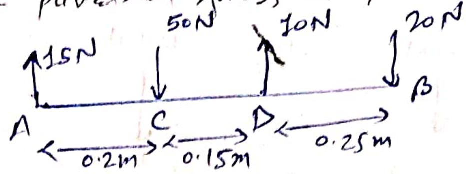
By Varignon's Theorem –

$\Sigma M_P = R \times x$

$1800 = 450 \times x$

$\Rightarrow \boxed{x = 4 \text{ cm}}$

Q. A rigid bar is subjected to a system of parallel forces as shown in Fig.
Reduce this system to —



(a) a single force
(b) A single force moment system at A
(c) A single force moment system at B.

(A) A single force or resultant —

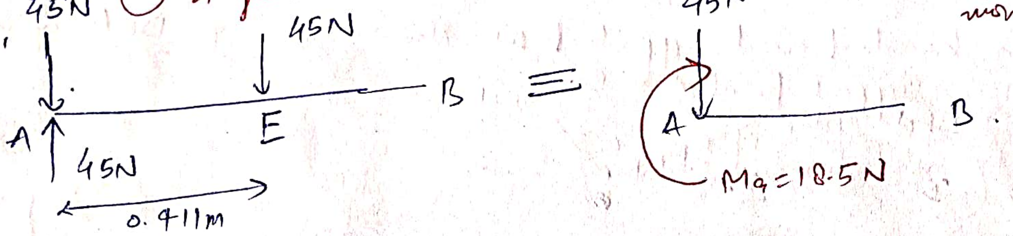
$\Rightarrow \uparrow \Rightarrow +ve \rightarrow \quad 15-50+10-20 \Rightarrow R=-45\,N \ (\downarrow)$

$\Sigma M_A = 50\times0.2 - 10\times0.35 + 20\times0.6$

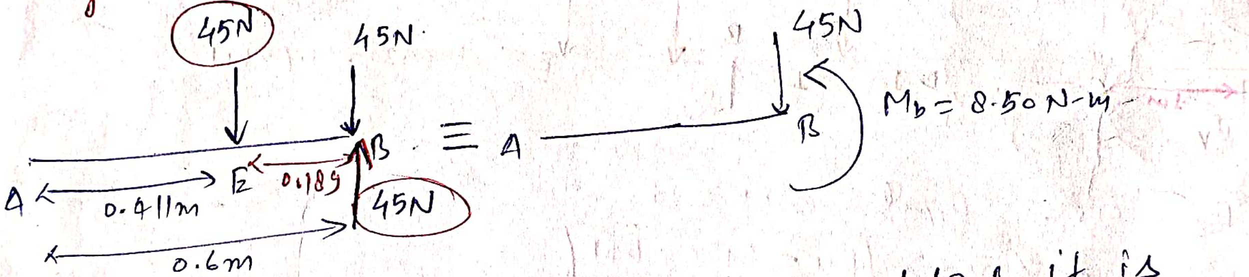
$\Sigma M_A = R \times x$ (Varignon's Theorem).

$x = \dfrac{18.5}{45} \Rightarrow \boxed{x = 0.411\,m}$

(b) Single force-moment at A $\Longrightarrow$ when a force of 45N acting at E shifted to A, it is accompanied by a moment



(c) Single Force-moment at B.

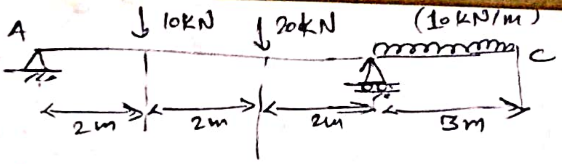


When the force of 45N acting at E is moved to B, it is accompanied by a anticlockwise moment

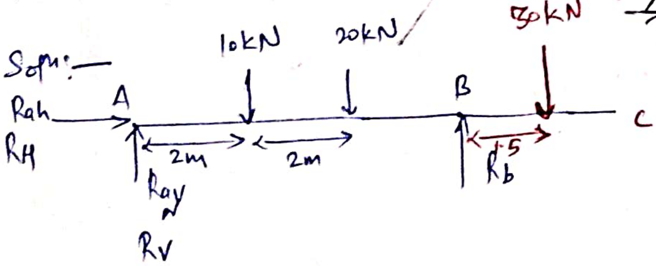
$M_b = -45\times0.189$ N-m

$= -8.5$ N-m

Q. Determine the reaction at A & B.



Soln:-



Applying Condition of Equilibrium -

ie - $\Sigma M = 0$, $\Sigma f_x = 0$, $\Sigma F_y = 0$

① $\Sigma f_x = 0 \Rightarrow R_{ah}$ or $R_H = 0$

② $\Sigma F_y = 0 \Rightarrow R_V - 10 - 20 + R_b - 30 = 0$ — (I)

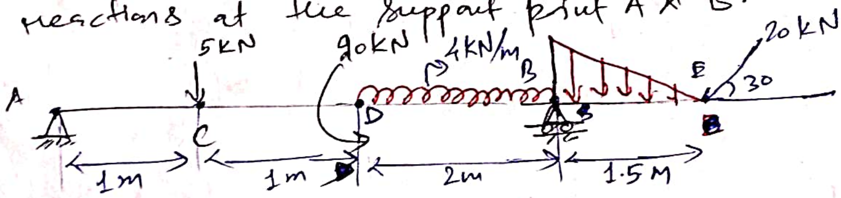
$\boxed{R_V = 5.83\ kN}$

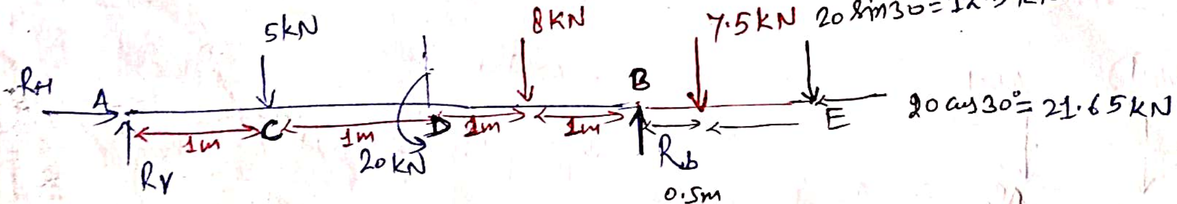
③ $\Sigma M_A \Rightarrow 0$

$10 \times 2 + 20 \times 4 - R_b \times 6 + 30 \times 7.5 = 0$

$\Rightarrow \boxed{R_b = 54.17\ kN}$

Q. A beam has been loaded and supported as shown in fig. Determine the reactions at the support point A & B.





$\Sigma f_x = 0$, $\Sigma f_y = 0$, $\Sigma M = 0$

$R_H - 21.65 = 0 \Rightarrow \boxed{R_H = 21.65\ kN}$

$\Sigma F_y = 0 \Rightarrow R_v - 5 - 8 + R_b - 7.5 - 12.5 = 0$

$R_v + R_b = 33$ — (I)

$\Sigma M_a = 0$ (+ve) $5 \times 1 - 20 + 8 \times 3 - R_b \times 4 + 7.5 \times 4.5 + 12.5 \times 5.5 = 0$

$\Rightarrow \boxed{R_b = 27.875\ kN}$

$\boxed{R_v = 5.125\ kN}$

Centre of Gravity, Centre of Mass & Centroid:—

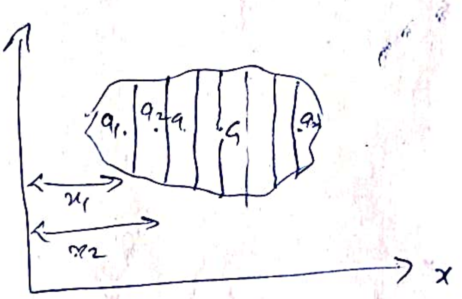
⇒ Centre of gravity of a body is defined as the point through which resultant of ~~the~~ gravitation force (weight) acts for any orientation of the body.

⇒ Centre of mass is the point where the entire mass of body is assumed to be concentrated.

° The weight of body is the product of $(m \times g)$ and if the small variation in 'g' from point to point on earth are neglected. then CM of body is same as its CG. ie CG and CM coinside

⇒ The plane figures, have only the length, area, and volume and no mass or weight. The point where the entire length, area or volume is assumed to be concentrated is called ~~the~~ centroid.

Location of Centroid / CG:—



Moment of areas of all the strips about y-axis –

$= a_1x_1 + a_2x_2 + a_3x_3 + \cdots$

$= \Sigma ax$

Moment of total Area A about the y-axis.

$= A\bar{x}$

$A \cdot \bar{x} = \Sigma ax$

$$\boxed{\bar{x} = \frac{\Sigma ax}{A}}$$

Similarly $$\boxed{\bar{y} = \frac{\Sigma a \cdot y}{A}}$$

The location of CG of solids.

$$\bar{x} = \frac{\Sigma mx}{\Sigma m} \quad \& \quad \bar{y} = \frac{\Sigma m \cdot y}{\Sigma m}$$

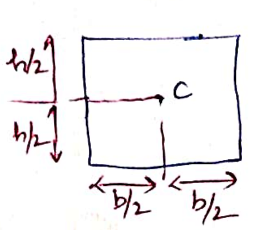
$m_1 = \rho \cdot V_1$, $m_2 = \rho V_2$ $m_3 = \rho \cdot V_3$ etc

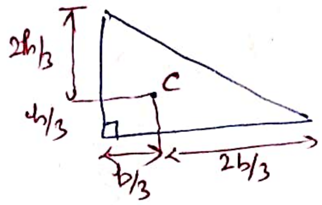
$V = V_1 + V_2 + V_3 + \text{———}$

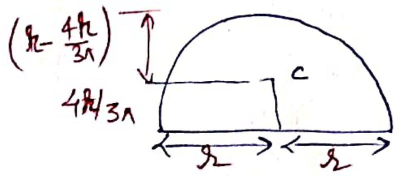
$$\bar{x} = \frac{\Sigma \rho \cdot Vx}{\Sigma \rho V} = \frac{\rho \Sigma V \cdot x}{\rho \Sigma V} \Rightarrow \frac{\Sigma Vx}{\Sigma V}$$

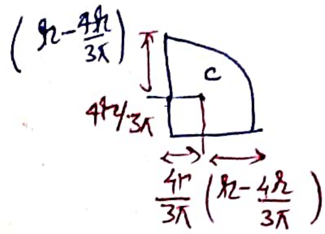
$$\boxed{\bar{y} = \frac{\Sigma V \cdot y}{\Sigma V}}$$

(1) Rectangle:- $\bar{x}$ (2) Right Angle Triangle:- (3) Semi Circle (4) Quarter Circle:-



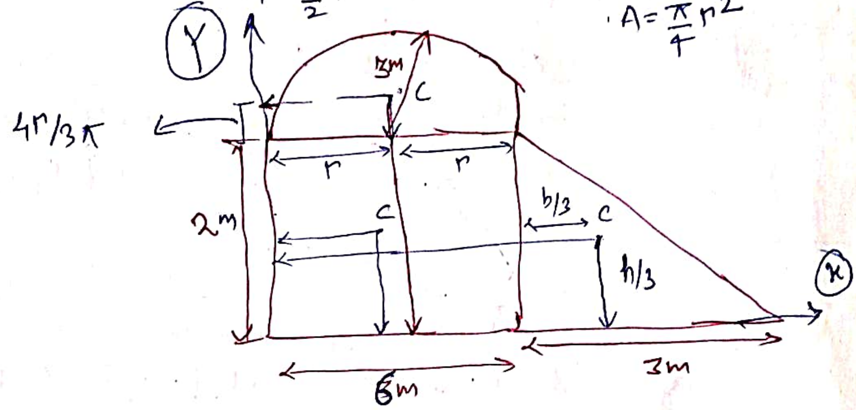


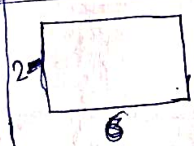
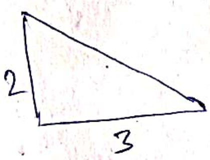
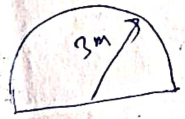




$A = b \times h$ $A = \frac{1}{2} \times b \times h$ $A = \frac{\pi}{2} r^2$ $A = \frac{\pi}{4} r^2$

Q. Find the centroid of given figure.



Sr. No	Shape	Area (m^2)	X (m)	Y (m)	A · x (m^3)	A · y (m^3)
1.		12	$b/2 = 3$	$h/2 = 1$	$12 \times 3 = 36$	$12 \times 1 = 12$
2.		$\frac{1}{2} \times 2 \times 3$ $= 3$	$6 + \frac{3}{3} = 7$	$\frac{h}{3} = \frac{2}{3}$ $= 0.666$	$3 \times 7 =$ 21	3×0.666 $= 1.998$
3.		$\frac{\pi}{2} r^2$ $\Rightarrow \frac{\pi}{2} \times 3^2$ $= 14.137$	$x = r$ $= 3$	$2 + \frac{4r}{3\pi}$ $= 2 + \frac{4 \times 3}{3\pi}$ $\Rightarrow 3.273$	14.137×3 $= 42.411$	14.137×3.273 $= 46.27$
		$\Sigma A = 29.137$			$\Sigma Ax = 99.411$	$\Sigma Ay = 60.268$

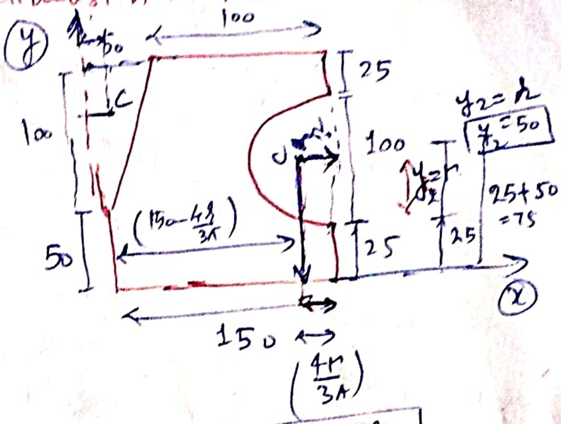
$$\bar{x} = \frac{\Sigma A x}{\Sigma A} = \frac{99.411}{29.137} \Rightarrow \bar{x} = 3.411 \, m$$

$$\bar{y} = \frac{\Sigma A \cdot y}{\Sigma A} = \frac{60.268}{29.137} \Rightarrow \bar{y} = 2.068 \, m$$

Centroid $= (\bar{x}, \bar{y})$

$= (3.411, 2.068)$

Q. Locate the centroid of the area shown in fig. All the dimension in mm.



a_1 = Rectangle ⇒

$a_1 = 150 \times 150 \Rightarrow 22500\ mm^2$

$x_1 = \frac{150}{2} = 75\ mm$

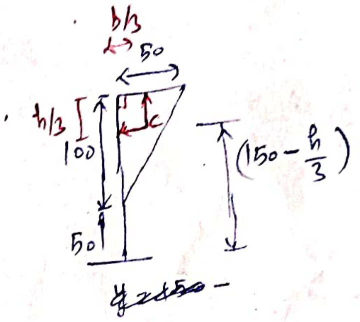
$y_1 = \frac{150}{2} = 75\ mm$

$\boxed{x_2 = 150 - \frac{4r}{3\pi}}$

Semi circle $a_2 = \frac{\pi}{2} r^2 = \frac{\pi}{2} \times 50^2 = 3925\ mm^2$

$x_2 = 150 - \frac{4r}{3\pi} \Rightarrow 150 - \frac{4 \times 50}{3\pi} \Rightarrow 128.77\ mm$

$y_2 = 25 + r \Rightarrow 25 + 50 = 75\ mm$



Triangle:— $a_3 = \frac{1}{2} \times b \times h = \frac{1}{2} \times 50 \times 100 = 2500\ mm^2$

$x_3 = \frac{b}{3} = \frac{50}{3} = 16.67\ mm$

$y_3 = 150 - \frac{h}{3} \Rightarrow 150 - \frac{100}{3} = 116.67\ mm$

$\bar{x} = \frac{a_1 x_1 - a_2 x_2 - a_3 x_3}{a_1 - a_2 - a_3} = \frac{22500 \times 75 - 3925 \times 128.77 - 2500 \times 16.67}{22500 - 3925 - 2500}$

$\boxed{\bar{x} = 70.94\ mm}$

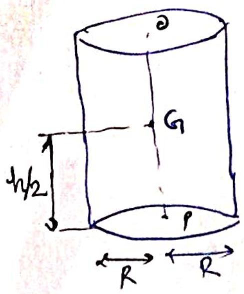
$\bar{y} = \frac{a_1 y_1 - a_2 y_2 - a_3 y_3}{a_1 - a_2 - a_3}$

$= \frac{22500 \times 75 - 3925 \times 75 - 2500 \times 116.67}{22500 - 3925 - 2500}$

$\boxed{\bar{y} = 68.52\ mm}$

$\boxed{\text{Centroid } C = 70.94,\ 68.52\ mm}$

(1) ~~Rectangle~~ Cylinder

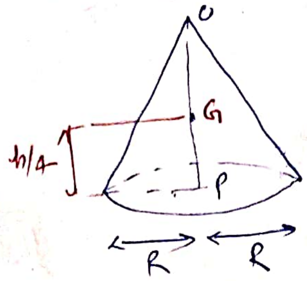


$x = R$

$y = h/2$

$V = \pi R^2 h$

(2) Cone:-



$x = R$

$y = h/4$

$V = \frac{1}{3}\pi R^2 h$

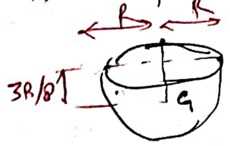
(3) Sphere:-

$x = R$

$y = R$

$V = \frac{4}{3}\pi R^3$

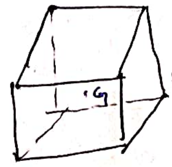
(4) Hemisphere:



$x = R$

$y = 3R/8$

$V = \frac{2}{3}\pi R^3$



$x = a/2$

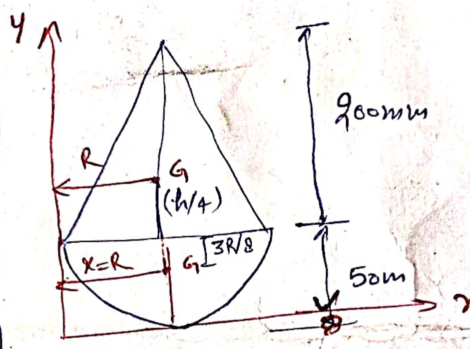
$y = a/2$

$V = a^3$

Q. A right circular cone of base diameter 100mm and height 200mm is place on the hemisphere of the same diameter. Calculate its centre of gravity.

Sl No	Solid	Volume (mm3)	x (mm)	y (mm)	V.x (mm4)	V.y
1.	Cone (200, 100)	5.235×10^5	50	100	26.17×10^6	52.35×10^6
2.	Hemisphere	2.617×10^5	50	31.25	13.089×10^6	81.81×10^5

$\Sigma V = 7.853\times10^5$ mm3 $\qquad \Sigma Vx = 39.269\times10^6 \qquad \Sigma Vy = 60.541\times10^6$



$$\bar{x} = \frac{\Sigma V.x}{\Sigma V} = \frac{39.269\times10^6}{7.85\times10^5} \Rightarrow \bar{x} = 50\,mm$$

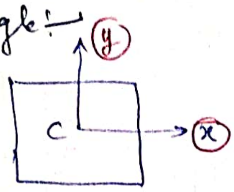
$$\bar{y} = \frac{\Sigma V.y}{\Sigma V} = \frac{60.541\times10^6}{7.85\times10^5} \Rightarrow 77.083$$

Centre of gravity $= G = (\bar{x}, \bar{y})$

$= (50, 77.083)$

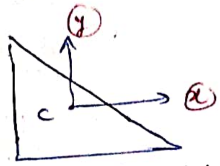
Moment of Inertia:-

(1) Rectangle:-



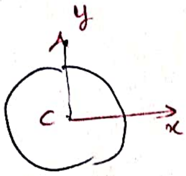
$$I_x = \frac{bh^3}{12}, \quad I_y = \frac{hb^3}{12}$$

(2) Triangle:-



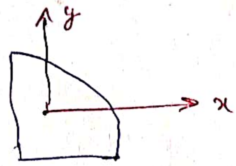
$$I_x = \frac{bh^3}{36}, \quad I_y = \frac{hb^3}{36}$$

(3) Circle:-



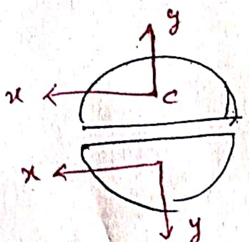
$$I_x = I_y = \frac{\pi}{4} r^4$$

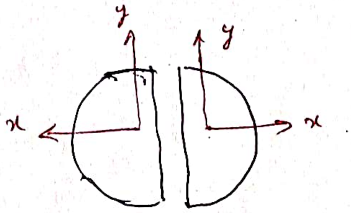
(4) Quarter Circle:-



$$I_x = I_y = \frac{\pi}{16} r^4$$

(5) Semi circle:-





$$I_x = I_y = \frac{\pi}{8} r^4$$

$$I_x = \int y^2 \, dA \quad \& \quad I_y = \int x^2 \, dA$$

Polar Moment of Inertia $\Rightarrow J_o = \int r^2 \, dA$

$$J_o = I_x + I_y$$

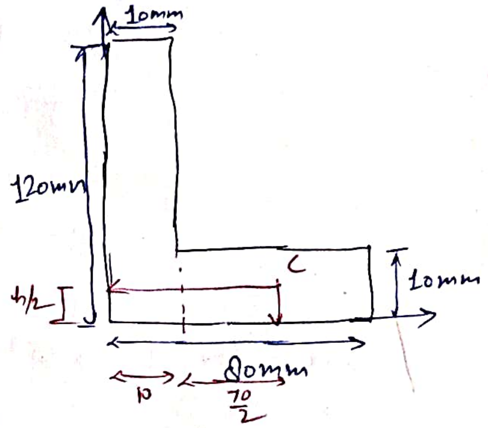
Moment of inertia of an Area = Area $\times$ (Distance)^2
= (Length)^4

Radius of gyration:-

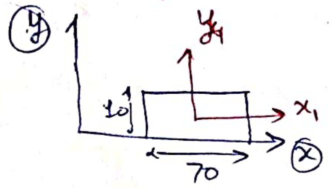
$$I_x = k_x^2 A$$

$$\boxed{k_x = \sqrt{\frac{I_x}{A}}}$$

Q. Find the M.O.I for the given L section about X and Y-axis.



(i) Rectangle (1):-



$A = 70 \times 10 \Rightarrow 700\,mm^2$

$X = 10 + \frac{70}{2} \Rightarrow X = 45\,mm, \quad y = \frac{10}{2} \Rightarrow y = 5\,mm$

$$I_{x_1} = \frac{bh^3}{12} = \frac{70 \times 10^3}{12} \Rightarrow \boxed{I_{x_1} = 5.833 \times 10^3\,mm^4}$$

$$I_{y_1} = \frac{hb^3}{12} = \frac{10 \times 70^3}{12} \Rightarrow \boxed{I_{y_1} = 285.833 \times 10^3\,mm^4}$$

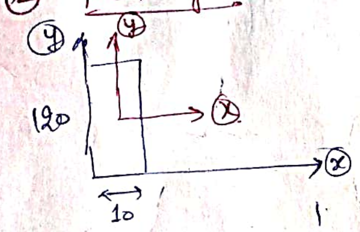
$$I_{xx_1} = I_x + A\,l^2 = 5.833 \times 10^3 + 700 \times 5^2$$

$$\boxed{I_{xx_1} = 23.333 \times 10^3\,mm^4}$$

$$I_{yy_1} = I_{y_1} + A\,l^2 = 285.833 + 700 \times 45^2$$

$$\boxed{I_{yy_1} = 1703.333 \times 10^3\,mm^4}$$

(2) Rectangle (2):-



$A = 1200\,mm^4$

$x = 5\,mm, \quad y = 60\,mm$

$$I_{x_2} = \frac{bh^3}{12} = \frac{10 \times 120^3}{12} \Rightarrow \boxed{I_{x_2} = 1.44 \times 10^6\,mm^4}$$

$$I_{y_2} = \frac{h \cdot b^3}{12} = \frac{120 \times 10^3}{12} \Rightarrow \boxed{I_{y_2} = 10 \times 10^3\,mm^4}$$

$$I_{xx_2} = I_{x_2} + A \times l^2 \Rightarrow 1.44 \times 10^6 + 1200 \times 60^2$$

$$\boxed{I_{xx_2} = 5.76 \times 10^6}$$

$$I_{yy_2} = I_{y_2} + A\,l^2 \Rightarrow I_{yy_2} = 10 \times 10^3 * 1200 \times 5^2 \Rightarrow \boxed{I_{yy_2} = 40 \times 10^3\,mm^4}$$

$$I_x = I_{xx_1} + I_{xx_2} \Rightarrow 23.333 \times 10^3 + 5.76 \times 10^6 \Rightarrow \boxed{I_x = 5.783 \times 10^6\,mm^4}$$

Ans

$$I_y = I_{yy_1} + I_{yy_2} \Rightarrow 1703.33 \times 10^3 + 40 \times 10^3 \Rightarrow \boxed{I_y = 1.7433 \times 10^6\,mm^4}$$

Direct:-

I_x = M.I of Rectangle (1) + M.I of Rectangle (2)

$$= I_{xx_1} + I_{xx_2}$$

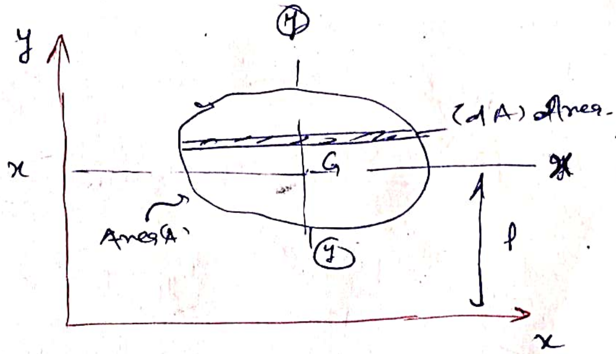
$$= (I_{x_1} + A\,l^2) + (I_{x_2} + A\,l^2)$$

$$= \left[\frac{70 \times 10^3}{12} + 70 \times 5^2\right] + \left[\frac{10 \times 120^3}{12} + 1200 \times 60^2\right]$$

$$\boxed{I_x = 5.783 \times 10^6\,mm^4}$$

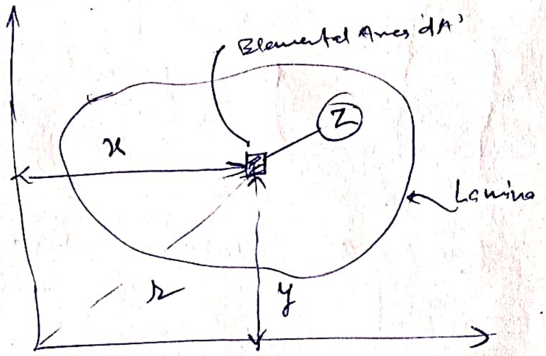
Parallel axis theorem:- The moment of inertia of a plane lamina about any axis is equal to the sum of its MOI about a parallel axis through its centre of gravity G and the product of its area (mass) and the square of the distance b/w. the two axes.

$$\boxed{I_{AB} = I_{xx} + A \times l^2}$$



Perpendicular axis Theorem:- The moment of inertia of a plane lamina about an axis ⊥r to the plane of the lamina is equal to the sum of the moment of inertia of the lamina about the two axes at the right angles to each other and intersecting each other at the point where the ⊥r axis passes through it.

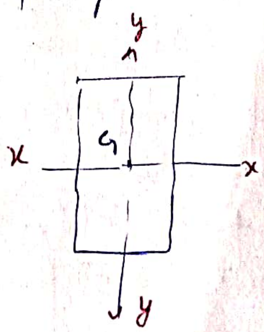
$$\boxed{I_{zz} = I_{xx} + I_{yy}}$$



Radius of gyration:- If the entire area (or mass) of a lamina is considered to be concentrated at a point such that there is no change in the moment of inertia about a given axis, then distance of that point from the given axis is called the radius of gyration.

$$I = A \times k^2$$

$$\boxed{k = \sqrt{I/A}}$$



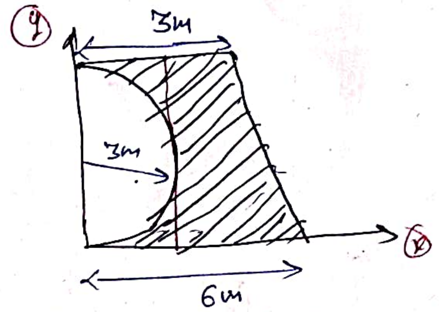
I_y = M.I of Rectangle 1 + M.I of Rectangle 2

$= (I_{y_1} + A \times l^2) + (I_{y_2} + A \times l^2)$

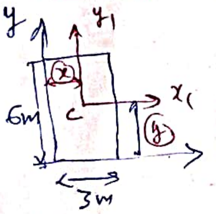
$= \left(\frac{10 \times 70^2}{12} + 700 \times 45^2\right) + \left(\frac{120 \times 10^3}{12} + 1200 \times 5^2\right)$

$\boxed{I_y = 1.7433 \times 10^6 \text{ mm}^4}$

Q. Find the M.I of shaded area shown in fig about x and y-axis.



(i) Rectangle:-



$A = 6 \times 3 \Rightarrow 18 \text{ m}^2$

$x = \frac{3}{2} = 1.5 \text{m}, \quad y = \frac{6}{2} = 3 \text{m}$

$I_{x_1} = \frac{bh^3}{12} = \frac{3 \times 6^3}{12} \Rightarrow 54 \text{ m}^4$

$I_{y_1} = \frac{hb^3}{12} = \frac{6 \times 3^2}{12} = 13.5 \text{ m}^4$

$I_{xx_1} = I_{x_1} + A \times l^2$

$= 54 + 18 \times 3^2$

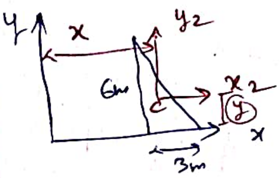
$\boxed{I_{xx_1} = 216 \text{ m}^4}$

$I_{yy_1} = I_{y_1} + A \times l^2$

$= 13.5 + 18 \times 1.5^2$

$\boxed{I_{yy_1} = 54 \text{ m}^4}$

(2) Triangle:-



$A = \frac{1}{2} \times 3 \times 6 \Rightarrow 9 \text{ m}^2$

$x = 3 + \frac{3}{3} \Rightarrow 4 \text{m}$

$y = \frac{6}{3} \Rightarrow 2 \text{m}$

$I_{x_2} = \frac{bh^3}{36} = \frac{3 \times 6^3}{36} \Rightarrow 18 \text{ m}^4$

$I_{y_2} = \frac{hb^3}{36} \Rightarrow \frac{6 \times 3^3}{36} \Rightarrow 4.5 \text{ m}^4$

$I_{xx_2} = I_{x_2} + A \times l^2$

$= 18 + 9 \times 2^2$

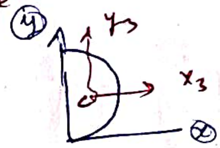
$\boxed{I_{xx_2} = 54 \text{ m}^4}$

$I_{yy_2} = I_{y_2} + A \times l^2$

$= 4.5 + 9 \times 4^2$

$\boxed{I_{yy_2} = 148.5 \text{ m}^4}$

(3) Semi circle:-



A =

$A = 14.137 \text{ m}^2$

$x = 1.273 \text{ m}$

$y = 3 \text{ m}$

$I_{x_3} = 0.392 \times r^4 = 0.392 \times 3^4$

$= 31.752 \text{ m}^4$

$I_{y_3} = 0.11 r^4 = 0.11 \times 3^4$

$= 8.91 \text{ m}^4$

$I_{xx_3} = I_{x_3} + A l^2$

$= 31.752 + 14.137 \times 3^2$

$\boxed{I_{xx_3} = 158.985 \text{ m}^4}$

$I_{yy_3} = I_{y_3} + A \times l^2$

$= 8.91 + 14.137 \times 1.273^2$

$\boxed{I_{yy_3} = 31.819 \text{ m}^4}$

$I_x = I_{xx_1} + I_{xx_2} - I_{xx_3}$

$= 216 + 54 - 158.985$

$\boxed{I_x = 111.015 \text{ m}^4}$

$I_y = I_{yy_1} + I_{yy_2} - I_{yy_3}$

$= 54 + 148.5 - 31.819 \text{ m}^4$

$\boxed{I_y = 170.681 \text{ m}^4}$

2nd method :-

I_x = M.I of Rectangle + M.I of Triangle − M.I of Semicircle

$= (I_x + A\ell^2) + (I_x + A\ell^2) - (I_x + A\ell^2)$

$= \left[\frac{3\times 6^3}{12} + 18\times 3^2\right] + \left[\frac{3\times 6^3}{36} + 9\times 2^2\right] - \left[0.392\times 3^4 + 14.137\times 3^2\right]$

$= 216 + 54 - 158.985$

$\boxed{I_x = 111.015\ m^4}$

I_y = M. of rectangle + M.I of triangle − M.I of semi circle

$= (I_y + A\ell^2) + (I_y + A\ell^2) - (I_y + A\ell^2)$

$= \left(\frac{6\times 3^2}{12} + 18\times 1.5^2\right) + \left(\frac{6\times 3^3}{36} + 9\times 4^2\right) - \left(0.11\times 3^4 + 14.137\times 1.273^2\right)$

$\boxed{I_y = 170.681\ m^4}$