Structure :— A framed structure is an assemblage of no. of bars or rods joined together in such a way to form a rigid framework; , this structure is designed to resist geometrical distortion under any applied system of loading. 3

Truss :— The structure is called a truss when the loads are applied only at joints and not any other point of the bar member.

Frame :— The structure is called a frame when some on its members are subjected to two or more forces. These forces are ~~not~~ applied not only at the joints but also at some other point of the member bars.

Plane frame & space frame :— In a plane frame all the members lie in a single plane and the forces act along the plane of the frame. Bridge trusses and roof trusses are the plane frame.

⇒ A frame in which all the members do not lie in the same plane is called a space frame. Tripod and suspension towers are the space frame.

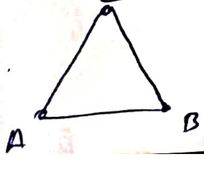
Statically determinate and statically Indeterminate frame :—

Statically determinate and statically The force analysis of the members of a statically determinate frame can be made by applying the equations of statics only. Equations of static equilibrium are not sufficient to determine the forces in statically indeterminate frames; there is need of considering their deformation also.

Perfect, deficient and redundant frames :— The structure is said to be perfect if the no. of members is just sufficient to prevent its distortion of shape when subjected to external loads.

For perfect Frame, $\boxed{m = 2J - 3}$

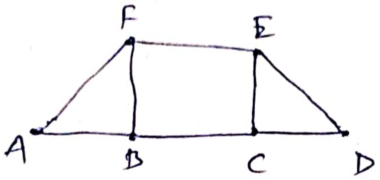
m = No. of member
J = No. of joint.



$m = 3, J = 3$
$m = (2J - 3)$
$\boxed{m = 3}$

1.

⇒ A structure is termed imperfect or deficient frame if the no. of member in it is less than that required for a perfect frame.



$\boxed{M < (2J-3)}$

$m = 8$

$J = 6$

$m = 2J - 3$

$= 2 \times 6 - 3$

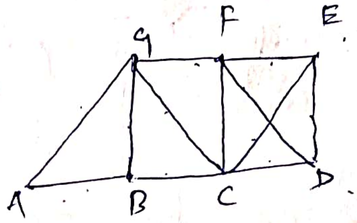
$\boxed{m = 9}$

⇒ A structure is termed redundant frame if no. of member in it is more than required for a perfect frame.

$m = 12$

$J = 7$

$\boxed{M > (2J-3)}$



$m = 2J - 3$

$= 2 \times 7 - 3$

$= 11$

Analysis of a Frame :- Assumptions —

(1.) The truss is a perfect one and statically determinate.

(2.) All the members comprising the truss are rigid and lie in the same plane.

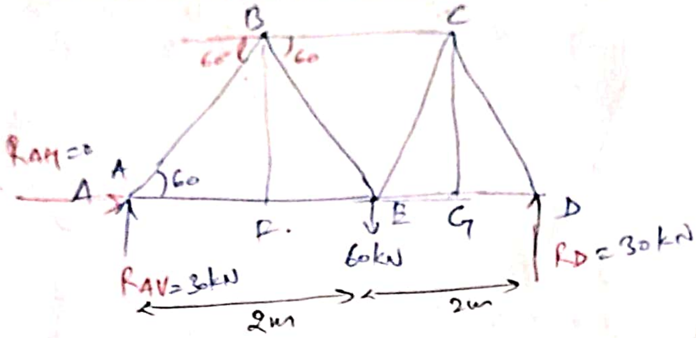
(3.) The members are slender and of uniform cross-section.

(4.) The external loads and reactions act at the joint only.

(5.) The self weight of the members is neglected because the self weight is small compared to loads they carry.

(6) The forces are transmitted from one member to another through smooth pins (no friction) fitting perfectly in the members.

Q Sol^n:-



$\overset{+}{\rightarrow} \Sigma F_x = 0$, $\boxed{R_{AH} = 0}$

$\Sigma F_y = 0$

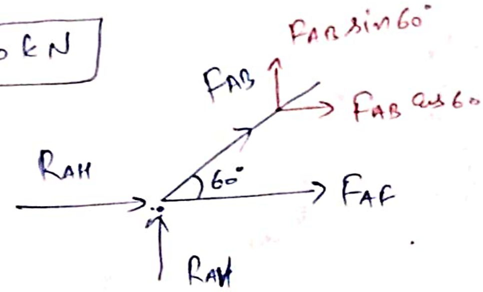
$R_{AV} + R_D - 60 = 0$

$R_{AV} + R_D = 60 \quad$ — ①

$\Sigma M_A = 0 \Rightarrow \quad 60 \times 2 - R_D \times 4 = 0$

$\boxed{R_D = 30\ kN}$ $\therefore$ $\boxed{R_{AV} = 30\ kN}$

F.B.D of Joint A:—



$+\uparrow \Sigma F_y = 0 \rightarrow$

$F_{AB}\sin 60 + 30 = 0$

$\boxed{F_{AB} = -34.641\ kN}$ (C)

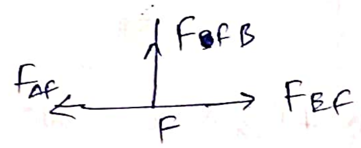
$\overset{+}{\rightarrow}$ $\Sigma F_x = 0$

$F_{AF} + F_{AB}\cos 60 = 0$

$F_{AF} - 34.641 \cdot \cos 60° = 0$

$\boxed{F_{AF} = 17.32\ kN}$ (T)

F.B.D of Joint F:—



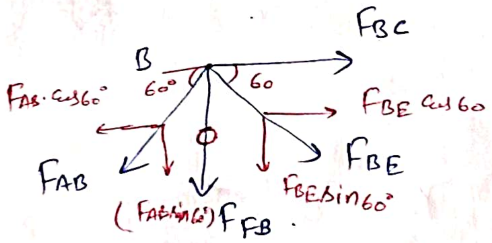
$\Sigma F_x = 0$

$F_{EF} - F_{AF} = 0$

$\boxed{F_{EF} = 17.32\ kN}$ (T)

$+\uparrow \Sigma F_x = 0 \Rightarrow$ $\boxed{F_{FB} = 0}$ (

F.B.D of Joint B:—



$\uparrow \Sigma F_y = 0$

$-F_{BE}\sin 60 - F_{AB}\sin 60 - F_{FB} = 0$

$-F_{BE}\sin 60 - (-34.641)\cdot\sin 60 - 0 = 0$

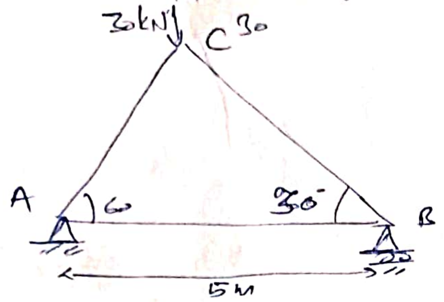
$\boxed{F_{BE} = 34.641\ kN}$ (T)

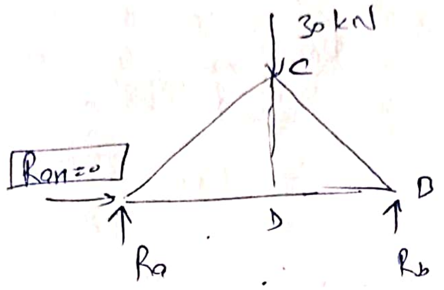
$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad F_{BC} + F_{BE}\cos 60 - F_{AB}\cos 60 = 0$

$F_{BC} + 34.641\cos 60 - (-34.641)\cdot\cos 60 = 0$

$\boxed{F_{BC} = -34.641\ kN}$ (C)

Q. Determine the forces in all the members of a truss with loading and support system shown in fig.





(FBD)

$AC = AB \cos 60$

$= 5 \times 0.5$

$AC = 2.5\,m$

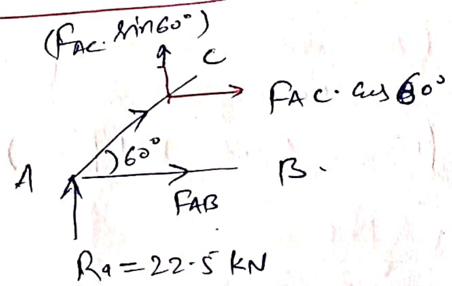
$AD = A.C \cos 60° \Rightarrow 2.5 \times 0.5$

$= 1.25\,m$

$\Sigma F_y = 0 - \quad R_a + R_b = 30\,kN$ —— (1)

$\Sigma M_A = 0 \quad R_b \times 5 = 30 \times 1.25 \Rightarrow \boxed{R_b = 7.5\,kN}$

$\boxed{R_a = 22.5\,kN}$



F.B.D of joint A :-

~~$\Sigma F_x = 0$~~,

$\Sigma F_y \Rightarrow 0$

$F_{AC} \sin 60 + \cancel{R_a} = 0$

$F_{AC} \sin 60 = -22.5\,kN$

$\boxed{F_{AC} = -25.97\,kN}$ (c)

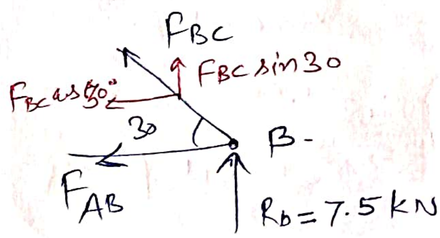
$\Sigma F_x = 0$

$F_{AC} \cos 60 + F_{AB} = 0$

$F_{AB} = -F_{AC} \cos 60$

$= -(-25.97) \cdot \cos 60$

$\boxed{F_{AB} = 12.99\,kN}$ (T)



F.B.D of Joint B:-

$\Sigma F_y = 0$

$F_{BC} \sin 30 + 7.5 = 0$

$F_{BC} = \dfrac{-7.5}{\sin 30}$

$\boxed{F_{BC} = -15\,kN \quad (c)}$

Sr. No	Force in Member	Magnitude	Nature
1.	FAB	12.99 kN	T
2.	FBC	15 kN	C
3	FAC	25.97	C

Due to symmetry of structure —

$F_{ED} = F_{AB}$

$F_{GD} = F_{AC}$

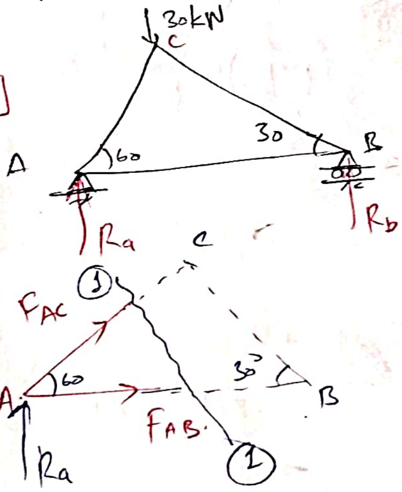
$F_{GE} = F_{EF}$

$F_{CG} = F_{FB}$

$F_{CE} = F_{FB}$

Sr. No	Force in Member	Magnitude	Nature
1.	F_{AB}, F_{CD}	34.641 kN	(C)
2.	F_{AF}, F_{GD}	17.32 kN	T
3.	F_{FB}, F_{CG}	0	—
4	F_{BE}, F_{CE}	34.641 kN	T
5.	F_{EF}, F_{GE}	17.32 kN	T
6	F_{BC}	34.641 kN	C

Q. Determine the forces in members AC and AB of a simple triangular truss with the loading & support system shown in adjoining figure. Use the method of section.



Soln:— $R_a = 22.5\,kN$

$R_b = 7.5\,kN$

$AC = AB\cos 60 \Rightarrow \boxed{AC = 2.5m}$

$BC = AB\sin 60 \Rightarrow \boxed{BC = 4.33\,m}$

$\Sigma M_b \Rightarrow 0$

$R_a \times AB + F_{AC} \times BC = 0$

$F_{AC} = R_A \times \dfrac{AB}{BC} = 22.5 \times \dfrac{5}{4.33}$

$\boxed{F_{AC} = 22.98\,kN\ (C)}$

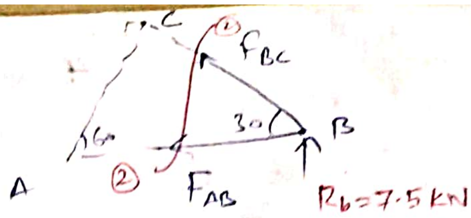
$\Sigma M_c = 0$

$R_a \times AC\cos 60 - F_{AB} \times AC\sin 60° = 0$

$F_{AB} = R_a \times \dfrac{AC\cos 60°}{AC\sin 60°}$

$\boxed{F_{AB} = 12.99\,kN\ (T)}$

To determine force in member BC –



$\Sigma M_a = 0$

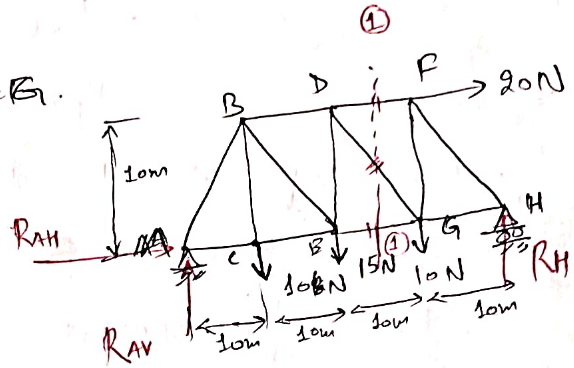
$-R_b \times AB - F_{BC} \times AC = 0$

$F_{BC} = -R_b \times \frac{AB}{AC}$

$F_{BC} = -7.5 \times \frac{5}{2.5}$

$\boxed{F_{BC} = -15\ kN\ (C)}$

Q. Find the force in member DF, DG, EG.



$\xrightarrow{+} \Sigma F_x = 0$

$R_{AH} + 20 = 0$

$\boxed{R_{AH} = -20\ N}$

$+\uparrow \Sigma F_y = 0$

$R_{AV} + R_H - 10 - 15 - 10 = 0$

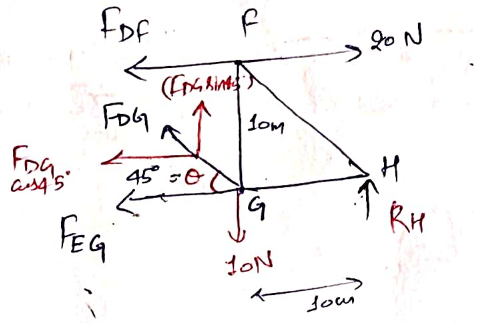
$R_{AV} + R_H = 35$ — (I) $\boxed{R_{AV} = 12.5\ N}$

$+\circlearrowright \Sigma M_A = 0$

$10 \times 10 + 15 \times 20 + 10 \times 30 + 20 \times 10 - R_H \times 40 = 0$

$\boxed{R_H = 22.5\ N}$

FBD of RHS part of section (1)(1)



$\tan\theta = \frac{10}{10}$

$\boxed{\theta = 45°}$

$+\uparrow \Sigma F_y = 0$

$F_{DG} \cdot \sin 45 - 10 + R_H = 0$

$\boxed{F_{DG} = -17.677\ N}$ (C)

$+ve\ \Sigma M_G = 0$

$-F_{DF} \times 10 + 20 \times 10 - 22.5 \times 10 = 0$

$\boxed{F_{DF} = -2.5\ N}$ (c)

Sr No	Force in Member	Magnitude	Nature
1	F_{DF}	2.5 N	(C)
2.	F_{DG}	17.677 N	(C)
3.	F_{EG}	35 N	(T)

$\xrightarrow{+} \Sigma F_x = 0$

$-F_{EG} - F_{DF} + 20 - F_{DG} \cos 45 = 0$

$-F_{EG} - (-2.5) + 20 - (-17.677) \cos 45 = 0$

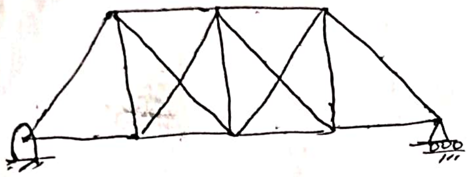
$\boxed{F_{EG} = 35\ N}$ (T)

Q. Examine the truss given in the adjoining diagrams-

(a)

(b)

(c)

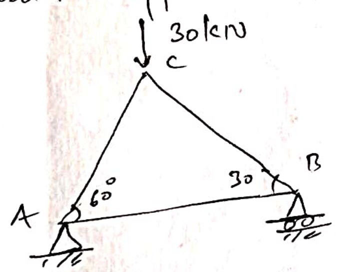


(a) $J = 8$, $m = 11$, $m = 2J-3 \Rightarrow 2\times8-3$
$11 \neq 13$ 'deficient frame'

(b) $J = 8$, $m = 13$, $m = 2J-3 \Rightarrow 2\times8-3$
$\boxed{m = 13}$ perfect truss

(c) $J = 8$, $m = 15$, ~~and~~ $m = 2J-3$
$\boxed{m > 13}$. over rigid or redundant.

Q. Determine the forces in all the members of a truss with the loading and support system shown in fig.



Friction:—

When a body slides over another body, a force is exerted at the surface of contact by the stationary body on the moving body. This resisting force is called the force of friction and acts in the direction opposite to the direction of motion.

Nature of friction:—

(1.) Dry friction:— Dry friction (also called coulomb friction) manifests when the contact surfaces are dry and there is tendency for relative motion.

Dry friction is further subdivided into—

(a.) Sliding Friction:— Friction b/w two surfaces when one surface slides over another.

(b.) Rolling Friction:— Friction b/w two surfaces which are separated by balls or rollers.

(2.) Fluid Friction:— Fluid friction manifests when a lubricating fluid is introduced b/w the contact surfaces of two bodies.

(3.) Static and Dynamic Friction:—

⇒ The static friction is the frictional force that develops b/w mating surfaces when subjected to external forces but there is no relative motion b/w them.

⇒ The dynamic friction is the frictional force that develops b/w mating surfaces when subjects to external forces and there is relative motion b/w them. The dynamic friction is also known as kinetic friction.

Laws of Solid Friction (Static or Dynamics):—

(1.) Friction acts tangential to the surfaces in contact and is in a direction opposite to that which motion is to impend i.e take place.

(2.) Friction force is max^m at the instant of impending motion. Its variation from zero to max^m value (limiting friction) depends upon the resultant force tending to cause motion.

(3.) The magnitude of limiting friction bears a constant ratio to the normal reaction b/w the mating surfaces.

(4.) Limiting friction is independent of area and shape of contact surfaces

(5.) Limiting friction Is depends upon the nature (roughness or smoothness) of the surface In contact

Coefficient of friction:— (μ):— The ratio of force of friction to the normal reaction b/w the contact surfaces.

$F \propto R$

$F = \mu \cdot R$

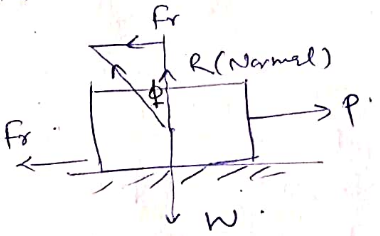
$$\boxed{\mu = F/R}$$

(i) when system is in state of impending motion, the frictional force has the limiting (max^m) value F_s and the ratio F_s/R is called the coefficient of static friction μ_s.

(ii) when motion starts, the max^m friction falls to the lower value F_k called kinetic friction. The ratio F_k/R is called the coefficient of kinetic friction μ_k.

$\mu_k < \mu_s$ for same pair of contact surfaces.

Angle of Friction:— The angle of friction (ϕ) is a measure of the limiting position of total reaction b/w the two contacting surfaces. It is defined as the angle which the resultant of normal reaction and limiting force of friction makes with the normal reaction.



F_r = frictional force

R = Normal Reaction

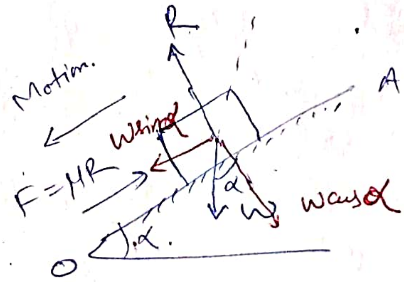
F = Limiting force of friction

S = Total reaction $S = \sqrt{R^2+F^2}$

$\tan\phi = \frac{F}{R}$ ↳ coefficient of friction μ.

$$\boxed{\mu = \tan\phi}$$

Angle of Repose:—



(1) Weight 'W' of block acting vertically downwards

(2) Normal Reaction 'R' acting at right angles to the inclined plane

(3) Limiting force of friction $F_r = \mu \cdot R$ acting up the plane as the block is to slide down

$\boxed{W\cos\alpha = R}$ —(I)

$\boxed{\mu R = W\sin\alpha}$ —(II)

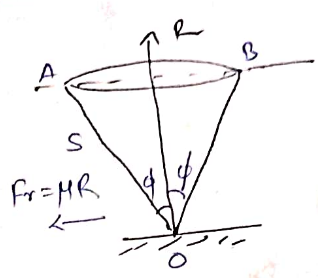
$\mu = \tan\alpha$

$\boxed{\tan\phi = \tan\alpha \text{ or } \phi = \alpha}$

The angle α of the inclined plane at which a block resting on it is about to slide down the plane is called the angle of Repose.

Cone of friction:-

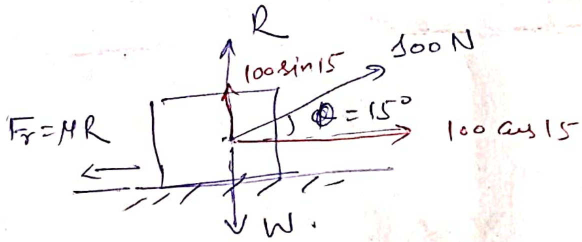
Total reaction $= \sqrt{F_r^2 + R^2}$



ϕ = angle with normal reaction

O is the vertex of a cone whose axis is R and semi-vertex angle is ϕ. Such a right circular cone is called the cone of friction.

Q. A body weighing 300 N is resting on a rough horizontal table. A pull of 100 N applied at the angle of 15° with the horizontal just cause the body to slide over the table. Make calculation for the normal reaction and the coefficient of friction.



Sol^n—

$\Sigma F_x = 0$

$F_r - 100 \cos 15 = 0$

$F_r = 100 \cos 15$

$\boxed{F_r = 96.59 \text{ N}}$

$\Sigma F_y = 0 \Rightarrow R + 100 \sin 15 - W = 0$

$\Rightarrow R = 300 - 100 \sin 15$

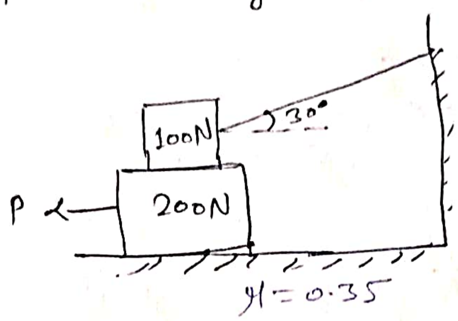
$\boxed{R = 274.12 \text{ N}}$

$\mu = F_r / R$

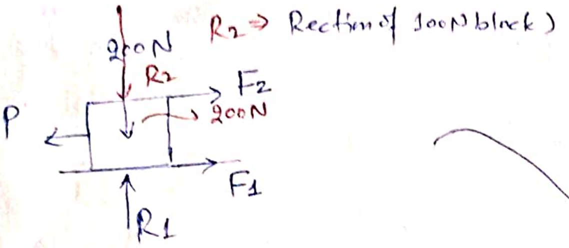
$= \dfrac{96.59}{274.12}$

$\boxed{\mu = 0.352}$

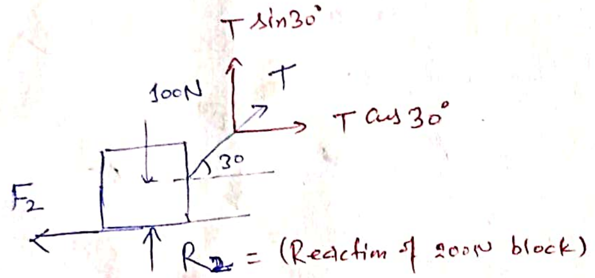
Q. A block of mass 200 N resting on a horizontal surface supports another block of 100 N as shown in figure. The smaller block is attach to the string from the wall. Find the horizontal force P just to move the 200 N block to the left. coefficient of friction is 0.35 for all rubbing surfaces.



Soln:– F.B.D of 200 N Block :–



F.B.D of 100 N Block :–



$\xrightarrow{+} \Sigma F_x = 0$

$T \cos 30 - F_2 = 0$

$T \cos 30 - \mu R_2 = 0$

$T \cos 30 - 0.35 R_2 = 0$ — (I)

$+\uparrow \Sigma F_y = 0$

$T \sin 30 + R_2 - 100 = 0$

$T \sin 300 + R_2 = 100$ — (II)

By eqn (I) & (II) —

$\boxed{T = 33.62 \text{ N}}$ & $\boxed{R_2 = 88.189 \text{ N}}$

$+\uparrow \Sigma F_y = 0$

$R_1 - R_2 - 200 = 0$

$R_1 - 88.189 - 200 = 0$

$\boxed{R_1 = 288.189 \text{ N}}$

$\xrightarrow{+} \Sigma F_x = 0$

$-P + F_1 + F_2 = 0$

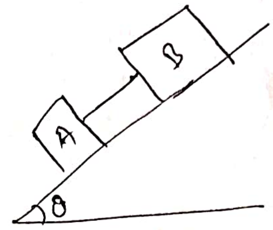
$-P + 0.3 R_1 + 0.3 R_2 = 0$

$P = 0.3(R_1 + R_2)$

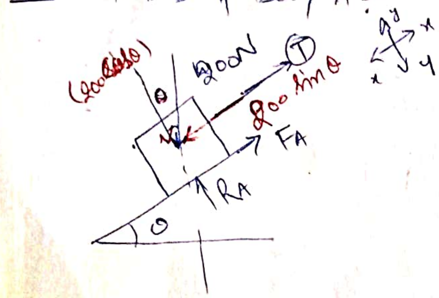
$= 0.3(288.189 + 88.189)$

$\boxed{P = 128.232 \text{ N}}$

Q. Bodies A & B are join by a chord parallel to the incline plane as shown in fig. For Body A $\mu = 0.2$ and weight = 200 N while for Body B $\mu = 0.5$ and weight = 300 N. Determine angle θ at which the motion impends. What is then the tension in the chord?



Sol^n:- F.B.D of Body A:-



$+\uparrow \Sigma F_y = 0$

$R_A - 200\cos\theta = 0$

$\boxed{R_A = 200\cos\theta}$ — (I)

$\xrightarrow{+} \Sigma F_x = 0$

$T + F_A - 200\sin\theta = 0$

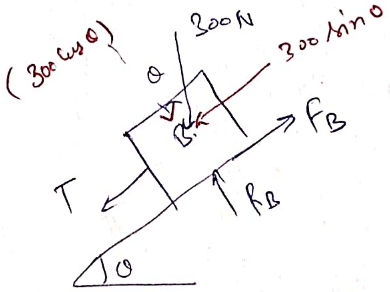
$T + \mu_A R_A - 200\sin\theta = 0$

$T + 0.2 \times 200\cos\theta - 200\sin\theta = 0$

$T + 40\cos\theta - 200\sin\theta = 0$

$\boxed{T = 200\sin\theta - 40\cos\theta}$ — (2)

F.B.D of Body B:-



$+\uparrow \Sigma F_y = 0$

$R_B - 300\cos\theta = 0$

$\boxed{R_B = 300\cos\theta}$ — (3)

$\xrightarrow{+} \Sigma F_x = 0$

$-T + F_B - 300\sin\theta = 0$

$-T + \mu \cdot R_B - 300\sin\theta = 0$

$-T + 0.5 \times 300\cos\theta - 300\sin\theta = 0$

$-T + 150\cos\theta - 300\sin\theta = 0$

$\boxed{T = 150\cos\theta - 300\sin\theta}$ — (4)

By eq^n (3) & (4)

$200\sin\theta - 40\cos\theta = 150\cos\theta - 300\sin\theta$

$\frac{\sin\theta}{\cos\theta} = \frac{190}{500}$

$\boxed{\theta = 20.806}$ Ans

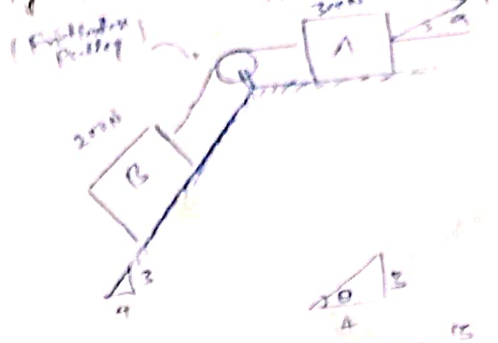
$T = 150 \cdot \cos 20.806 - 300\sin 20.806$

~~θ = 20.80°~~

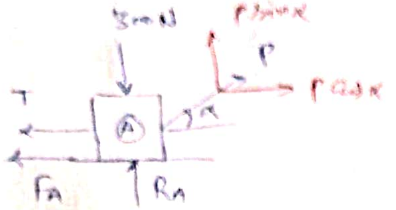
$\boxed{T = 33.649\ N}$ Ans.

Q. Find the least value of force P that will just start the system of blocks as shown in fig. moving to the right. The coefficient of friction under each block is 0.3.

Sol^n:—



F.B.D of Block A



$\tan\theta = \frac{3}{4}$

$\theta = 36.869°$

$\xrightarrow{+} \Sigma F_x = 0$

$P\cos\alpha - T - F_A = 0$

$P\cos\alpha - T - 0.3 \times R_A = 0$ — (I)

$+\uparrow \Sigma F_y = 0$

$P\sin\alpha + R_A - 300 = 0$

$R_A = 300 - P\sin\alpha$

By putting R_A value in eq^n - (I)

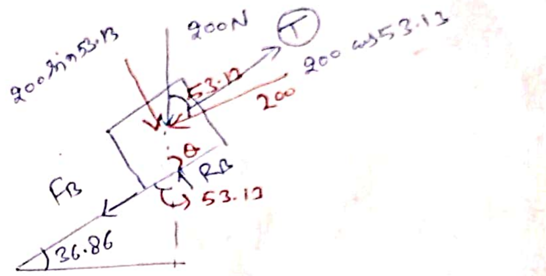
$P\cos\alpha - T - 0.3(300 - P\sin\alpha) = 0$

$P\cos\alpha - T - 90 + 0.3P\sin\alpha = 0$

$P(\cos\alpha + 0.3\sin\alpha) = T + 90$

$P = \dfrac{T + 90}{\cos\alpha + 0.3\sin\alpha}$

F.B.D of Block B:—



$+\uparrow \Sigma F_y = 0$

$R_B - 200\sin 53.13 = 0$

$\boxed{R_B = 160N}$

$\xrightarrow{+} \Sigma F_x = 0$, $T - F_B - 200\cos 53.13 = 0$

$T - 0.3 \times 160 - 200\cos 53.13 = 0$

$\boxed{T = 168\ N}$

$\therefore P = \dfrac{168 + 90}{\cos\alpha + 0.3\sin\alpha}$

$P = \dfrac{258}{\cos\alpha + 0.3\sin\alpha}$ — (II)

For P_{min} the value of denominator will be max^m.

$\frac{d}{d\alpha}(\cos\alpha + 0.3\sin\alpha)$

$\Rightarrow -\sin\alpha + 0.3\cos\alpha = 0$

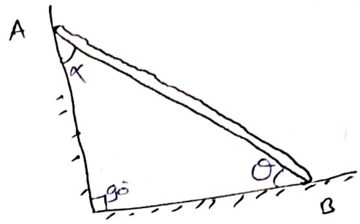
$\tan\alpha = 0.3 \Rightarrow$ $\boxed{\alpha = 16.7°}$

By eq^n (II) —

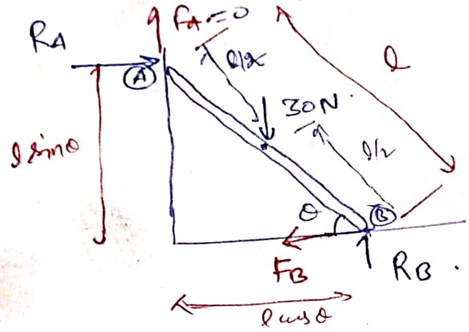
$P = \dfrac{258}{\cos 16.7 + 0.3 \times \sin 16.7}$

$\boxed{P = 243.12}$

Q. One end of a uniform ladder weights 30 N rest against a smooth wall and other end on the rough horizontal floor. coefficient of friction is 0.24 Find the Inclination of ladder to the horizontal when it is on the point of slipping.



Sol^n:- F.B.D of Ladder:-



$+\uparrow \Sigma F_y = 0$

$R_B - 30 = 0$

$\boxed{R_B = 30 \text{ N}}$

Frictional Force $= \mu \cdot R_B$

$= 0.24 \times 30$

$\boxed{F_B = 7.2 \text{ N}}$

$\xrightarrow{+} \Sigma F_x = 0$

$R_A - F_B = 0$

$\boxed{R_A = 7.2 \text{ N}}$

$+\circlearrowleft \Sigma M_B = 0$

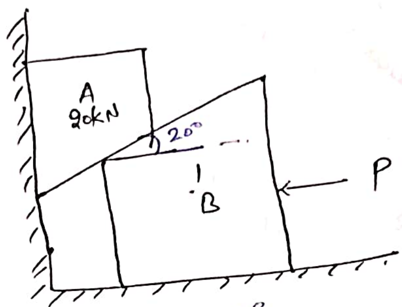
$R_A \times l \sin\theta - 30 \times \frac{l}{2} \cos\theta = 0$

$7.2 \times l \sin\theta - 15 \cdot l \cdot \cos\theta = 0$

$\frac{\sin\theta}{\cos\theta} = \frac{15}{7.2}$

$\boxed{\theta = 64.358^\circ}$

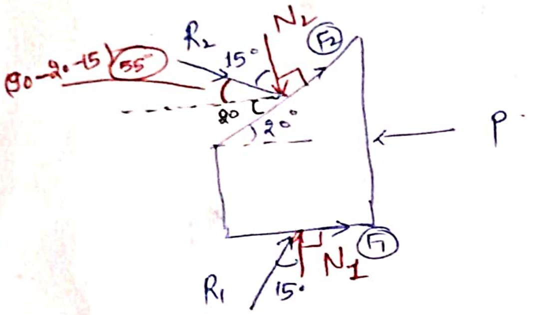
Q. A weight of 20 kN is to be raised by means of wedge. Find the value of P for impending motion of block A. Angle of friction for the all the contact surface is 15°.

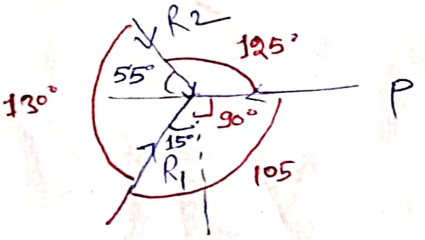


$\phi = 15^\circ$

Soln:—

F.B.D of wedge:—



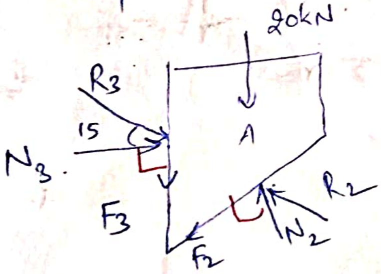


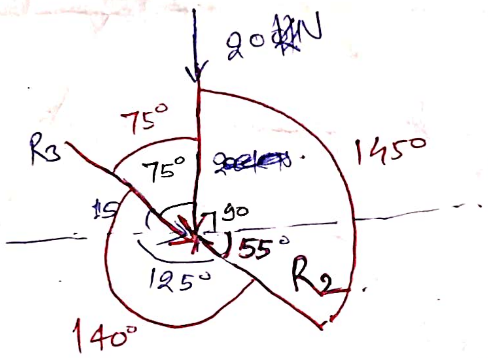
By Lami Theorem—

$$\frac{R_1}{\sin 125} = \frac{R_2}{\sin 105} = \frac{P}{\sin 130}$$

$$\frac{30.054}{\sin 105} = \frac{P}{\sin 130} \Rightarrow \boxed{P = 23.834\ N}$$

F.B.D of Block A





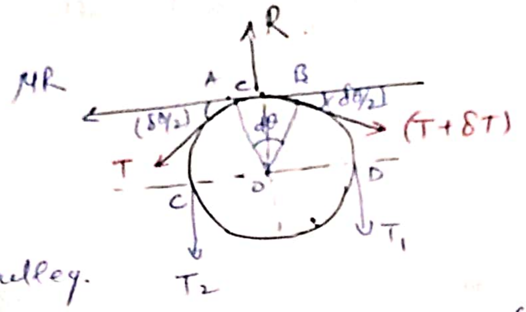
By Lami Theorem —

$$\frac{R_2}{\sin 75} = \frac{20}{\sin 140} = \frac{R_3}{\sin 145}$$

$$R_2 = \frac{20 \sin 75}{\sin 140}$$

$$\boxed{R_2 = 30.054\ N}$$

Friction in Flat Belt:—



Let — T_1 = Tension on tight side of the belt

T_2 = Tension on slack side of belt

θ = Angle of contact of belt with the pulley

μ = coefficient of friction b/w the belt & pulley.

The belt portion AB is in equilibrium under the action of following force—

1. Tension T in belt at A
2. Tension $(T+\delta T)$ in belt at B
3. Normal Reaction R at C
4. Frictional force, μR

Resolving the force—

$\Sigma F_x \Rightarrow$ $\mu R + T\cos\frac{\delta\theta}{2} = (T+\delta T)\cdot\cos\frac{\delta\theta}{2}$

Since $\delta\theta$ is very so—

$$\cos\frac{\delta\theta}{2} = 1$$

$$\mu\cdot R + T = T + \delta T$$

$$\mu R = \delta T \quad \text{— (I)}$$

$\Sigma F_y \Rightarrow 0$ $\quad R = T\sin\frac{\delta\theta}{2} + (T+\delta T)\sin\frac{\delta\theta}{2}$

$\delta\theta$ is very small

$$\sin\frac{\delta\theta}{2} = \frac{\delta\theta}{2}$$

$$R = T\cdot\frac{\delta\theta}{2} + T\cdot\frac{\delta\theta}{2} + \frac{\delta T\cdot\delta\theta}{2}$$

$\because$ $\frac{\delta T\cdot\delta\theta}{2}$ is very small, it can be neglected

$$R = T\cdot\delta\theta \quad \text{— (II)}$$

From ~~the~~ value of R from (ii) in eqn (I). we get—

$$\mu\cdot R = \delta T$$

$$\mu(T\cdot\delta\theta) = \delta T$$

$$\frac{\delta T}{T} = \mu\cdot\delta\theta$$

Integrating b/w the tension limit $\quad \int_{T_2}^{T_1}\frac{dT}{T} = \int_0^{\theta}\mu\cdot d\theta$

$$\Rightarrow \boxed{T_1/T_2 = e^{\mu\theta}}$$

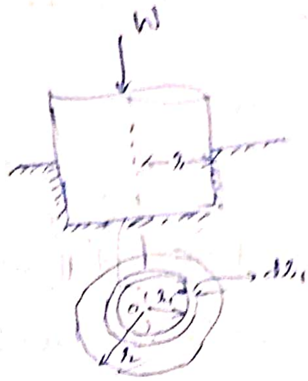
Friction in Flat Pivot and collar Bearing:—

Let W = Total load over the bearing surface

p = intensity of pressure

r = radius of pivot

μ = coefficient of friction.



Consider the elemental ring of radius r_1 and thickness δr_1

(a) Considering Uniform pressure—

Area of elemental ring $= 2\pi r_1 \cdot \delta r_1$

Load on the ring = pressure × Area

$$\delta W = p \times 2\pi r_1 \cdot \delta r_1$$

Total load on the ring $W = \int_0^r 2p\pi r_1 \, dr_1$

$$W = \pi \times p \cdot r^2$$

$$p = \frac{W}{\pi r^2} \quad \text{—(I)}$$

Frictional resistance on the elemental ring

$$\delta F_r = \mu \cdot \delta W \Rightarrow \mu (2\pi r_1 dr_1) p.$$

— Frictional torque on the ring —

$$T_r = \delta F_r \times r_1$$

$$= \mu \times 2\pi r_1 dr_1 \times r_1$$

$$= \mu \cdot 2\pi r_1^2 dr_1$$

Total frictional torque

$$T = \int_0^r \mu \cdot 2\pi r_1^2 dr_1$$

$$= 2 \times \pi p \cdot \frac{r^3}{3}$$

$$T = \frac{2}{3}\pi \mu \cdot p r^3$$

$$T = \frac{2}{3}\pi \mu \cdot \frac{W}{\pi r^2} \cdot r^3$$

$$\checkmark \boxed{T = \frac{2}{3}\mu \cdot W \cdot r}$$

Power lost in friction $P = T \times \omega$

$$\boxed{P = \frac{2}{3}\mu \cdot W \cdot r \cdot \omega}$$

(b) Considering uniform wear:

$P \cdot r_1 = \text{Constant} = C$

$p = \frac{C}{r_1}$

Load taken by the ring.

$\delta W = p \times \text{Area of the ring}$

$= p \times 2\pi r_1 \, dr_1 = \frac{C}{r_1} \times 2\pi r_1 \, dr_1$

$= 2\pi C \, dr_1$ — (I)

Total load taken by the ring $W = \int_0^{r} 2\pi C \, dr_1 = 2\pi C r$

(or) $C = \frac{W}{2\pi r}$ — (II)

Frictional torque on ring element is given by —

$T_r = \mu \cdot \delta W \cdot r_1 = \mu \, (2\pi C \, dr_1) \cdot r_1$

$= 2\pi \mu \, C r_1 \, dr_1$

$T = \int_0^{r} 2\pi \mu \, C \cdot r_1 \, dr_1$

$= 2\pi \mu \cdot C \frac{r^2}{2}$

Substituting the value of constant C from eq^n — (II)

$T = 2\pi \mu \cdot \frac{W}{2\pi r} \cdot \frac{r^2}{2}$

$$\boxed{T = \frac{1}{2} \mu \cdot W \cdot r}$$

Power lost is given —

$P = T \cdot \omega$

$$\boxed{P = \frac{1}{2} \mu \cdot W \cdot r \cdot \omega}$$

Flat collar Bearing:—

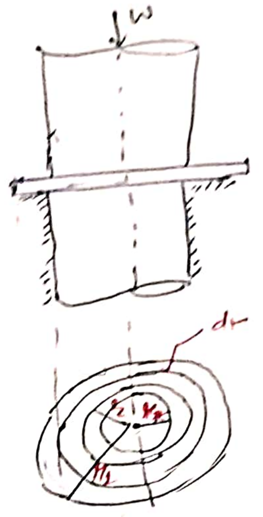
Let r_1 = External radius of the collar

r_2 = Internal radius of the collar

p = Intensity of press^n

W = Axial load

Consider an elemental ring of radius r & the thickness δr.



Area of the elemental ring = $2\pi r\,\delta r$

Load on the ring $dW = p \times 2\pi r\, dr$

Total Axial load $W = \int_{r_2}^{r_1} p \cdot 2\pi r\, dr$

Frictional torque on the ring

$$T_r = \mu \cdot \delta W \cdot r$$
$$= \mu \cdot p \cdot 2\pi r \cdot dr \cdot r$$

Total frictional torque

$$T = \int_{r_2}^{r_1} \mu \cdot p \cdot 2\pi \cdot r\, dr \cdot r$$
$$= \int_{r_2}^{r_1} \mu \cdot 2\pi\, p\, r^2\, dr$$

(a) considering uniform press^n—

Area of Bearing surface $A = \pi(r_1^2 - r_2^2)$

Uniform intensity of press^n $p = \dfrac{W}{\pi(r_1^2 - r_2^2)}$ —(I)

$$T = \int_{r_2}^{r_1} \mu \cdot 2\pi \times p\, r^2\, dr$$
$$= 2\pi\mu \cdot p \left[\frac{r^3}{3}\right]_{r_2}^{r_1}$$
$$= \frac{2}{3}\pi\mu \cdot p \cdot (r_1^3 - r_2^3)$$

substituting p from eq^n (I) —

$$T = \frac{2}{3}\pi\mu \cdot \frac{W(r_1^3 - r_2^3)}{\pi(r_1^2 - r_2^2)}$$

$$\boxed{T = \frac{2}{3}\mu \cdot W\left[\frac{r_1^3 - r_2^3}{r_1^2 - r_2^2}\right]}$$

considering Uniform wear:—

$$p \cdot r = \text{constant} = c$$

$$\text{so} — \quad p_1 r_1 = p_2 r_2 = c$$

Total load on the bearing —

$$W = \int_{r_2}^{r_1} p \cdot 2\pi r \, dr = \int_{r_2}^{r_1} (p r) \, 2\pi \, dr = \int_{r_2}^{r_1} c \cdot 2\pi \, dr$$

$$W = c \cdot 2\pi (r_1 - r_2)$$

$$c = \frac{W}{2\pi (r_1 - r_2)} \quad — \text{(I)}$$

Total frictional torque on the bearing —

$$T = \int_{r_2}^{r_1} \mu \cdot W \cdot r$$

$$= \int_{r_2}^{r_1} \mu \cdot c \; 2\pi \cdot r \cdot dr$$

$$= 2\pi \mu c \left[\frac{r^2}{2} \right]_{r_2}^{r_1}$$

$$= \frac{2\pi \mu c (r_1^2 - r_2^2)}{2}$$

Substituting the value of c from eqn (I) —

$$T = \pi \cdot \mu (r_1^2 - r_2^2) \times \frac{W}{2\pi (r_1 - r_2)}$$

$$= \frac{\mu \cdot W}{2} \frac{(r_1 - r_2)(r_1 + r_2)}{(r_1 - r_2)}$$

$$= \mu \cdot W \frac{(r_1 + r_2)}{2}$$

$$\boxed{T = \mu \cdot W \cdot r_m}$$

r_m = mean radius

~~Flat~~ collars