Unit - III

Kinematics:— is concerned with the description of motion of objects independent of causes of motion.

Kinetics:— relates to action of forces and the resulting motion.

Types of Motion:—

(1) Rectilinear Motion:— when a particle moves along a path which is a straight line, it is called rectilinear motion.

(2) Curvilinear Motion:— when a particle moves along a curved path it is called curvilinear motion. If the curved path lies in a plane it is called plane curvilinear motion.

Displacement, Velocity and Acceleration:—

⇒ The change of position of a particle or a body with respect to a certain fixed reference point is termed as displacement.

⇒ The rate of change of position of a body with respect to time is called velocity.

⇒ The rate of change of velocity of a body with respect to time is called acceleration.

$$V_{av} = \frac{\Delta x}{\Delta t}$$

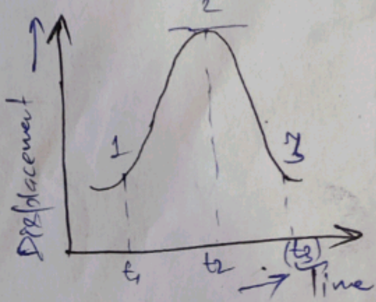
Instantaneous velocity $v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$

Average acceleration $a = \frac{\Delta v}{\Delta t}$

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Distance - time Graph (x-t curve):—

⇒ The slope of x-t curve at any instant gives the velocity of the body at that instant.



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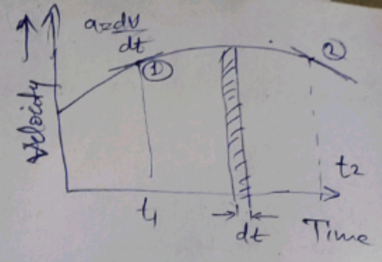
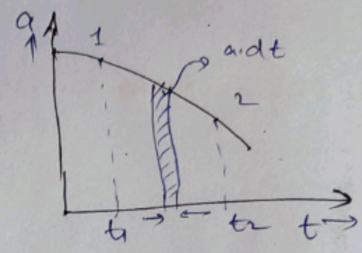
Velocity-time Graph (v-t curve):—



⇒ The area under the v-t curve corresponding to a given time interval gives the change in displacement or the distance travelled by the body during the same time interval.

Acceleration-time graph (a-t curve):—



⇒ The area under a-t curve corresponding to a given time interval gives the change in velocity of the body during the same time interval.

Eqn of Rectilinear Motion:—

When a body moves in a straight line with uniform acceleration the eqn of motion are—

$$v = u + at, \quad v^2 = u^2 + 2as, \quad s = ut + \frac{1}{2} \cdot a t^2$$

Distance travelled in 'n'th second

$$S_n = u + \frac{a}{2}(2n-1)$$

Q. A vehicle having rectilinear motion is moving with a velocity of 36 km/hr and accelerates uniformly to 72 km/hr over a distance of 200m. Calculate the acceleration and the time taken. How much distance will be covered by the vehicle in the 5th second?

Solⁿ:— Given Data:—

$$u = 36 \frac{km}{hr} \Rightarrow 36 \times \frac{5}{18} = 10\ m/sec$$

$$v = 72 \frac{km}{hr} = 72 \times \frac{5}{18} = 20\ m/sec.$$

If a = acceleration & t = time taken

$$v^2 = u^2 + 2a.s$$

$$20^2 = 10^2 + 2 \times a \times 200$$

$$\Rightarrow \boxed{a = 0.75\ m/s^2}$$

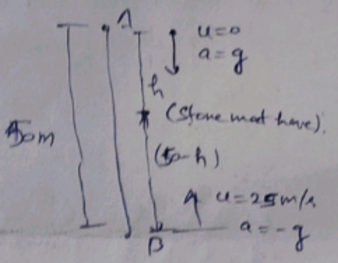
$$v = u + a \cdot t$$

$$20 = 10 + 0.75 \times t \Rightarrow \boxed{t = 13.33\ sec}$$

$$S_n = u + \frac{a}{2}(2n-1) \Rightarrow 10 + \frac{0.75}{2}(2 \times 5 - 1) \Rightarrow \boxed{S_5 = 13.375\ m}$$

Q. A stone is dropped from the top of tower 50 m high. At the same time another stone is thrown up from the foot of the tower with a velocity of 25 m/s. At what distance from the top and after how much time the two stones cross each other?

Solⁿ:— Let the two stone cross each other at distance = h from top of tower after 't' sec.



$$s = ut + \frac{1}{2}at^2 \quad \text{— (A)}$$

For downward motion of First stone

$u = 0, \quad s = h, \quad a = g = 9.81\,m/s^2$

$$h = 0 \times t + \frac{1}{2} \times 9.81 \times t^2$$
$$= 4.905\,t^2 \quad \text{— (1)}$$

For upward motion of IInd stone —

$u = 25\,m/s, \quad s = (50 - h), \quad a = -g = -9.81\,m/s^2$

By eqⁿ (A) —

$$\therefore (50 - h) = 25 \times t + \frac{1}{2} \times (-9.81) \times t^2$$
$$(50 - \cancel{h}) = 25t - \cancel{4.905t^2} \quad \text{— (II)}$$
$$\cancel{h} = \cancel{4.905\,t^2} \quad \text{— (I)}$$
$$50 = 25t \Rightarrow \boxed{t = 2\,sec}$$
$$h = 4.905 \times (2)^2 \Rightarrow 19.62\,m$$

Ans.

Q. A particle starts with velocity V_0, Its acceleration and velocity are related by the eqⁿ. $a = -kv$ where k = constant, v is the velocity of the particle, a is the acceleration of particle. Find the displacement time relation.

Solⁿ:—

$$a = -kv.$$
$$a = \frac{dv}{dt} = -kv.$$
$$\int \frac{dv}{v} = -k\int dt$$
$$\ln v = -k.t + C \quad \text{— (1)}$$

At $t = 0, \quad v = V_0$

$$\ln V_0 = -k \times 0 + C$$
$$C = \ln V_0$$
$$\ln v = -kt + \ln V_0$$
$$\ln \frac{v}{V_0} = -kt$$
$$\frac{v}{V_0} = e^{-kt}$$

$$v = V_0 \cdot e^{-kt}$$
$$\frac{dx}{dt} = V_0 \cdot e^{-kt}$$
$$\int dx = \int V_0 \cdot e^{-kt}\,dt$$
$$x = V_0 \cdot \frac{e^{-kt}}{-k} + C$$

At $t = 0, \ x = 0$

$$0 = +\frac{V_0}{k}$$
$$x = -\frac{V_0}{k}e^{-kt} + \frac{V_0}{k}$$
$$\boxed{x = \frac{V_0}{k}\cdot\left(1 - e^{-kt}\right)}$$

Ans

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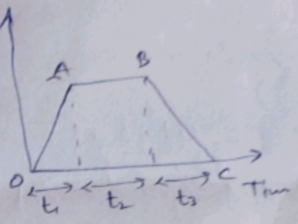
Kinetics

Q. A train starts from rest and increases its speed from 0 to v m/s with a constant acceleration a_1 m/s², runs at this speed for some time and finally comes to rest with a constant deceleration a_2 m/s². If the total distance travelled is 'x' meter, find the total time 't' required for this journey.

Solⁿ :—



Portion OA,

$V = u + at$

$V = 0 + a_1 \times t_1$

$t_1 = V/a_1$ — (1)

Portion BC :— $V = u + at$

$0 = V - a_2 t_3 \Rightarrow t_3 = \frac{V}{a_2}$ — (2)

Total distance travelled by the train (x) is given by the area under the velocity-time graph =

$$x = \frac{1}{2} \times (V \times t_1) + V \times t_2 + \frac{1}{2}(V \times t_3)$$

$$\frac{x}{V} = \frac{t_1}{2} + t_2 + \frac{t_3}{2}$$

$$t_2 = \frac{x}{V} - \frac{t_1}{2} - \frac{t_3}{2} \quad \text{— (3)}$$

Total time of travel $t = t_1 + t_2 + t_3$

$$t = \frac{V}{a_1} + \left(\frac{x}{V} - \frac{t_1}{2} - \frac{t_3}{2}\right) + \frac{V}{a_2}$$

$$= \frac{x}{V} + \frac{V}{a_1} - \frac{V}{2a_1} - \frac{V}{2a_2} + \frac{V}{a_2}$$

$$= \frac{x}{a} + \frac{V}{a_1} - \frac{V}{2a_1} + \frac{V}{2a_2}$$

$$\boxed{t = \frac{x}{V} + \frac{V}{2}\left(\frac{1}{a_1} + \frac{1}{a_2}\right)}$$

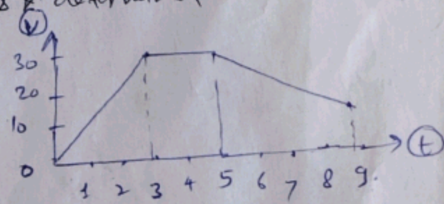
Q. A car moving in a straight line starts from rest, accelerates uniformly at 10 m/s² for the first 3 s, then travels at uniform speed for the next 2 s and finally uniformly (decelerates) at 5 m/s² for next 4 s. Draw the acceleration time, velocity-time graphs & determine the distance travelled by the car during the given time period.

Solⁿ :—

$V = u + at$

V after 3 second, $V = 0 + 10 \times 3 \Rightarrow V = 30$ m/s

V after 9 sec. $V = 30 + (-5) \times 4 = 10$ m/sec.



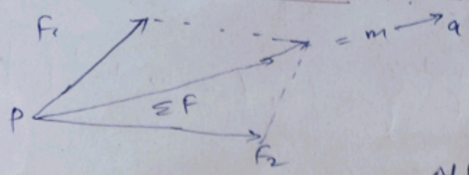
Area under v-t curve gives—

Distance travelled $= \left(\frac{1}{2} \times 3 \times 30\right) + (30 \times 2) + \left(\frac{30+10}{2}\right) \times 4$

$= 185$ m Ans

:- Kinetics :-

Eqⁿ of Rectilinear Motion:-



Consider a particle P of mass m having an acceleration 'a' when acted upon by several forces (say F_1 & F_2). Let ΣF be the resultant of these force -

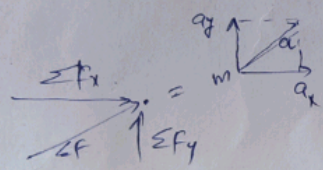
Applying Newton Second Law

$$\Sigma F = ma$$

Where the acceleration 'a' of the particle is in the direction of resultant force ΣF.

ω

$$\left.\begin{aligned} \Sigma F_x &= m\cdot a_x \\ \Sigma F_y &= m\cdot a_y \end{aligned}\right\} \text{Eq}^n \text{ of motion of the particle.}$$



Eqⁿ of Dynamic equilibrium "D'Alembert's principle:-

The eqⁿ of motion of particle P, -

$$\Sigma F = ma$$

$$\Sigma F - ma = 0$$

which means that the resultant of the external force ΣF and the force $(-ma)$ is zero. The force $(-ma)$ is called Inertia force.

$$\Sigma F + \underbrace{(-m\cdot a)}_{\text{Inertia force}} = 0$$

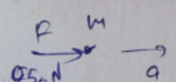
or in component form.

$$\left.\begin{aligned} \Sigma F_x + \underbrace{(-m\cdot a_x)}_{I.F} &= 0 \\ \Sigma F_y + (-m\cdot a_y) &= 0 \end{aligned}\right\} \text{Eq}^n \text{ of dynamic equilibrium of the particle.}$$

- So to write the eqⁿ of dynamic equilibrium of a particle add a fictious force equal to the inertia force to the external force acting on the particle and equate to sum (resultant) to zero. This concept of is known as D'Alembert's Principle.

Q. A force of 250N acts on a body of mass (m = 100kg). Find the acceleration of body.

Solⁿ -



$$\Sigma F = m\cdot a$$

$$250 = 100 \times a$$

$$\boxed{a = 2.5\,m/s^2}$$

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Q.1. Stone is dropped into a well in heard to strike the water after 4 sec. Find the depth of well, if the velocity of sound is 350 m/sec.

Solⁿ:- Let h = depth of well

t_1 = Time taken by stone to strike the water

t_2 = Time " by sound to reach from surface of water to top of well

$$t_1 + t_2 = 4 \quad \text{— (I)}$$

Considering downward motion —

$$h = ut + \frac{1}{2} g t^2$$

$$h = 0 + \frac{1}{2} \times 9.81 \times t^2$$

$$h = 4.905\, t_1^2$$

Considering the motion of sound, time taken by sound to reach from surface of water to top of well is —

$$\boxed{\because V = \frac{d}{T}}$$

$$t_2 = \frac{\text{Depth of well}}{\text{Speed of sound}} = \frac{h}{350}$$

$$t_2 = \frac{4.905\, t_1^2}{350}$$

$$t_1 + t_2 = 4$$

$$t_1 + \frac{4.905\, t_1^2}{350} = 4$$

$$4.905 t_1^2 + 350 t_1 = 1400$$

$$4.905 t_1^2 + 350 t_1 - 1400 = 0$$

$$t_1 = 3.79 ✓, \ -75.15 ✗$$

$$h = 4.905 \times (3.79)^2$$

$$\boxed{h = 70.45 \text{ m}} \quad \text{Ans}$$

Q. A particle moves along a straight line with an acceleration prescribed by the relation: $a = (4t^2 - 3t + 2)$ where a is in m/s^2, and 't' is in second. The particle has a velocity of 10 m/s at $t = 3$ sec. and it is located 12 m to the right of origin at $t = 2$ sec. Determine the position and velocity of the particle after 5 second.

Solⁿ:-

$$\frac{dv}{dt} = a = 4t^2 - 3t + 2$$

$$v = \frac{4 \times t^3}{3} - \frac{3}{2} t^2 + 2t + c$$

At $t = 3$ sec, $V = 10$ m/s,

$$10 = \frac{4}{3} \times (3)^3 - \frac{3}{2} \times (3)^2 + 2 \times (3) + c$$

$$c = -18.5$$

$$\boxed{v = \frac{4t^3}{3} - \frac{3}{2}t^2 + 2t - 18.5}$$

$$\frac{dx}{dt} = V = \frac{4}{3}t^3 - \frac{3}{2}t^2 + 2t - 18.5$$

$$x = \frac{4}{3} \times \frac{t^4}{4} - \frac{3}{2} \times \frac{t^3}{3} + \frac{2t^2}{2} - 18.5t + c$$

$$x = \frac{t^4}{3} - \frac{1}{2}t^3 + t^2 - 18.5t + c$$

At $t = 2$, $x = 12$

$$12 = \frac{(2)^4}{3} - \frac{(2)^3}{3} + (2)^2 - 18.5 \times (2) + c$$

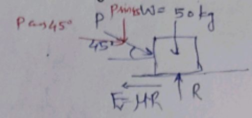
$$\Rightarrow \boxed{c = 43.67}$$

$$x = \frac{t^4}{3} - \frac{1}{3}t^3 + t^2 - 18.5t + 43.67$$

at $t = 5$: $x = 122$ m, $V = 120.67$ m/s Ans

Q A body of mass 50 kg rests on a rough horizontal surface ($\mu = 0.4$) and is acted upon a push applied at an angle of 45° to the horizontal. Determine the magnitude of push if it causes the body to move with an acceleration of 2 m/s².

Soln:—



$R = W + P \sin 45°$

$R = 50 \times 9.81 + P \cdot \sin 45°$

$R = 0.707P + 490.5$

Frictional force $F_r = \mu \cdot R$

$= 0.4(0.707P + 490.5)$

$= 0.2828P + 196.2$

Applying Newton's second law of motion,

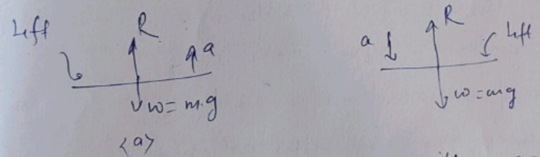
$F = m \cdot a$

$P\cos 45° - \mu R = m \cdot a$

$P\cos 45° - (0.2828P + 196.2) = 50 \times 2$

$\boxed{P = 698.25\ N}$

Motion of Lift:— consider a person of mass m inside a lift. The weight 'mg' of the person acts vertically downwards while the reaction R acts vertically upwards:



Case 1:— when the lift moves upwards with an acceleration a, the force causing motion is $(R - mg)$.

$\therefore R - mg = ma$.

$R = m\left(1 + \frac{a}{g}\right)$ — I

Case 2:— when the lift moves downwards with acceleration a, the force causing motion is $(mg - R)$.

$\therefore mg - R = m \cdot a$

$R = mg\left(1 - \frac{a}{g}\right)$ — II

If Tension T in the cable supporting the load.

$T = W\left(1 + \frac{a}{g}\right)$, when lift moves upwards.

$T = W\left(1 - \frac{a}{g}\right)$ when lift moves downwards

Momentum:— is the product of mass and velocity of a body and represents the energy of motion stored in a moving body.

Force $\propto$ Rate of change of (mass × velocity)

$\propto$ Mass × rate of change of velocity

$F \propto$ mass × acceleration

$F \propto m \cdot a$

$F = \frac{m \cdot a}{g_c}$ $\qquad g_c$ = proportionality constant

" The change in linear momentum per unit time is proportional to the impressed force and takes place in the direction of the force.

$$F \times t = m \cdot V_f - m \cdot V_i = m(V_f - V_i)$$

Law of conservation of momentum:— Total momentum of any group of objects always remain the same if no external force acts on it.

Consider that a body A of mass m_1 moving with velocity u_1 collides with another body B of mass m_2 and moving with velocity u_2. Let V_1 & V_2 be their velocities after the collision. Then —

momentum of masses before collision $= m_1 u_1 + m_2 u_2$

Momentum of masses after collision $= m_1 V_1 + m_2 V_2$

In accordance with law of conservation of momentum

momentum before collision = momentum after collision

$$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$$

Impulse = change of momentum $\qquad$ Impulse-momentum

= mass × change of velocity

Q. Determine the force necessary to produce an acceleration of $4\,m/s^2$ in a mass of 250 kg.

Solⁿ:— From Newton's second law, $F = m \cdot a$

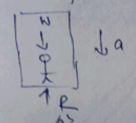
$F = 250 \times 4 = 1000\,N$

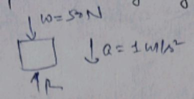
or

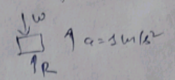
$F = \frac{250 \times 4}{9.81} \doteq 101.94\ kg_f$ (Gravitational units)

Q. An elevator has a downward acceleration of 1 m/s². What press^n will be transmitted to the floor of elevator by a man weighing 500N travelling in the lift? Find the press^n if elevator had an upward acceleration of 1 m/s².

Sol^n:—







Downward Motion:—

$\Sigma F = m.a \Rightarrow m.a = W - R \Rightarrow \frac{W}{g} a = W - R$

$R = W\left(1 - \frac{a}{g}\right) = 500\left(1 - \frac{1}{9.81}\right)$

$\boxed{R = 449 N}$

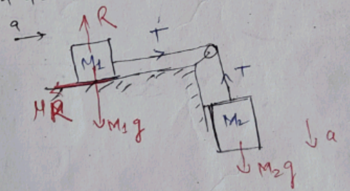
Upward motion:— $m.a = R - W \Rightarrow \frac{W}{g} = R - W$

$R = W\left(1 + \frac{a}{g}\right)$

$= 500\left(1 + 1/9.81\right)$

$\boxed{R = 550.97 N}$

Q. Two blocks of masses M_1 & M_2 are connected by a flexible but inextensible string as shown in fig. Assuming coefficient of friction b/w block M_1 and the horizontal surface to be μ. Find the acceleration of the masses and tension in the string. Assume $M_1 = 10kg$, $M_2 = 5kg$ & $\mu = 0.25$



Sol^n:— Let M_2 moves down with acceleration of a m/s². The acceleration of M_1 is same as M_2.

For Block M_1:—

$\Sigma F_x = m.a_x$ & $\Sigma F_y = m.a_y$

$M_1 a = T - \mu.R$ $\quad$ $0 = R - M_1 g$

$10 \times a = T - 0.25 \times 98.1$ $\quad$ $R = 10 \times 9.81$

$10a = T - 24.525$ —(I) $\quad$ $\boxed{R = 98.1 N}$

For Block M_2:—

$\Sigma F_y = 0 \Rightarrow M_2 . a = M_2 g - T$

$5a = 5 \times 9.81 - T$

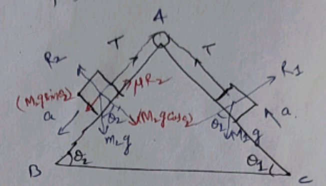
$T = 5(a + 9.81)$ —(II)

By eq^n (I) & (II) $\boxed{a = 1.635 \ m/s^2}$

$\boxed{T = 40.875 N}$ — Answer

Motion of two bodies connected over inclined planes:—

Q. Two blocks of masses M_1 & M_2 are placed on two incline plane of elevation θ_1 and θ_2 and are connected by a string as shown in fig. Find the acceleration of the masses. The coefficient of friction b/w the block and the plane is μ.



Given Data:— $M_1 = 5kg$ $M_2 = 10kg$

$\theta_1 = 30°$, $\theta_2 = 60°$ & $\mu = 0.33$

Solⁿ: Motion of Mass M_2:—

$\Sigma f_x = m \cdot a_x$

$M_2 \cdot a = M_2 g \sin\theta_2 - \mu R_2 - T$

$10 \times a = 10 \times 9.81 \times \sin 60 - 0.33 \times R_2 - T$ — ⓐ $\Rightarrow 10a = 84.95 - 16.18 - T$ — ⓐ

$\boxed{10a = 68.77 - T}$ — (I)

$\Sigma f_y = m \cdot a_y$ $\{a_y = 0$

$0 = R_2 - M_2 g \cos\theta_2$

$R_2 = M_2 \cdot g \cos\theta_2 \Rightarrow 10 \times 9.81 \times \cos 60$

$\boxed{R_2 = 49.05\ N}$

Motion of Mass M_1:—

$\Sigma f_x = M \times a_x \Rightarrow M_1 a = T - M_1 g \sin\theta_1 - \mu R_1$

$5 \times a = T - 5 \times 9.81 \times \sin 30 - 0.33 R_1$

$5a = T - 24.52 - 14.01$

$\boxed{5a = T - 1051}$ — (II)

$\Sigma f_y = m \cdot a_y \Rightarrow 0 = R_1 - M_1 g \cos\theta_1$

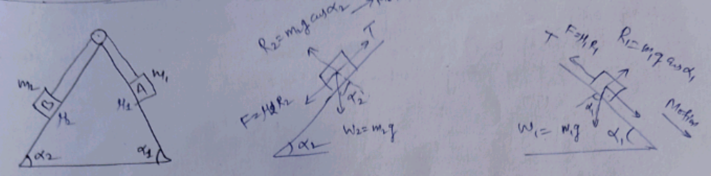
$R_1 = M_1 g \cos\theta_1 = 5 \times 9.81 \times \cos 30$

$\boxed{R_1 = 42.47\ N}$

By eqⁿ (I) & (II) —

$\boxed{a = 2.014\ m/s^2}$ Ans

Motion of two bodies connected over inclined planes:—



$$m_1 g \sin\alpha_1 - T - \mu_1 m_1 g \cos\alpha_1 = m_1 a \quad \text{— (I)}$$

$$T - m_2 g \sin\alpha_2 - \mu_2 m_2 g \cos\alpha_2 = m_2 a \quad \text{— (II)}$$

$$a = \frac{W_1 \sin\alpha_1 - W_2 \sin\alpha_2 - \mu_1 W_1 \cos\alpha_1 - \mu_2 W_2 \cos\alpha_2}{(W_1 + W_2)} \times g$$

$$T = \frac{g\, m_1 \cdot m_2}{(m_1 + m_2)} \left[(\sin\alpha_1 + \sin\alpha_2) - (\mu_1 \cos\alpha_1 - \mu_2 \cos\alpha_2) \right]$$

$$T = \frac{W_1 W_2}{(W_1 + W_2)} \left[(\sin\alpha_1 + \sin\alpha_2) - (\mu_1 \cos\alpha_1 - \mu_2 \cos\alpha_2) \right]$$

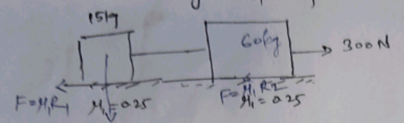
For smooth horizontal surfaces, $\mu_1 = \mu_2 = 0$

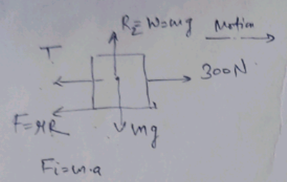
$$a = \frac{W_1 \sin\alpha_1 - W_2 \sin\alpha_2}{(W_1 + W_2)} \cdot g$$

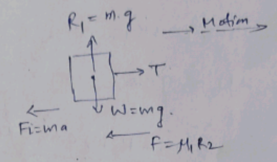
$$T = \frac{W_1 \cdot W_2}{(W_1 + W_2)} (\sin\alpha_1 + \sin\alpha_2)$$

Pressure on pulley $R = 2T \sin\frac{(\alpha_1 + \alpha_2)}{2}$

Q. Two blocks of mass 60 kg and 15 kg are connected by a string and move along a rough horizontal surface. When a force of 300 N is applied to the block of 60 kg mass as shown in fig. Apply D' Alembert's principle to determine the acceleration of the blocks and tension in the string. Presume that coefficient of friction b/w the sliding surface of blocks and the plane is 0.25.







⇒ Since ~~the~~ system moves right, friction force acts towards left

F_i need to be applied towards left to bring the system in a state of dynamic equilibrium

$$F = \mu_1 R_1 + \mu_2 R_2 \qquad (\mu_1 = \mu_2 = 0.25)$$
$$= 0.25\,(R_1 + R_2) \Rightarrow 0.25\,(m_1 g + m_2 g)$$
$$= 0.25 \times 9.81\,(15 + 60)$$
$$F = 183.94\ N$$

$$F_i = m.a \Rightarrow (m_1 + m_2)a \Rightarrow (15 + 60)a \Rightarrow 75\,a$$

Eqn for dynamic equilibrium (D' Alembert's principle) –

$$300 - 183.94 - 75a = 0$$
$$\boxed{a = 1.547\ m/s^2}$$

(ii) Considering dynamic equilibrium of block of mass 60 kg

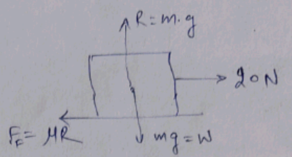
$$300 - T - F_i - F = 0$$
$$300 - T - 60 \times 1.547 - 0.25 \times (60 \times 9.81)$$
$$\boxed{T = 60.30\ N}$$

The tension 'T' in the string could also be worked out by considering dynamic equilibrium of the block of 15 kg mass.

$$\Sigma F_x = 0$$
$$T - F_i - F = 0$$
$$T - 15 \times 1.547 - 0.25 \times (15 \times 9.81) = 0$$
$$\boxed{T = 5998\ N}$$

Q. A body of 5 kg mass is initially at rest on a rough horizontal surface ($\mu = 0.2$) and is acted upon by a 20 N pull applied horizontally. Calculate —
(a) The W/D by the net force on the body in 5 seconds.
(b) Change in K.E of the body in 5 second

Sol. —



$R = m.g$
$= 5 \times 9.81$
$= 49.05 \text{ N}$

$F_f = \mu . R$
$= 0.2 \times 49.05$
$= 9.81 \text{ N}$

Net force causing motion $= 20 - 9.81 \text{ N}$
$= 10.19 \text{ N}$

Applying Newton's IInd law of motion
$F = m.a$
$10.19 = 5 \times a$
$\boxed{a = 2.038 \text{ m/s}^2}$

$S = ut + \frac{1}{2} a t^2$, Distance travelled in 5 sec.
$= 0 + \frac{1}{2} \times 2.038 \times (5)^2$
$S = 25.475 \text{ m}$

$W/D = F \times S$
$= 10.19 \times 25.479$
$W/D = 259.59 \text{ N-m}$

(b) The body starts from rest ($u = 0$) and accordingly initial K.E of body is zero
$K.E_1 = 0$

Final $K.E_2 = \frac{1}{2} m . v^2$

$v = u + a.t$
$= 0 + 2.038 \times 5$
$v = 10.19 \text{ m/s}$

$K.E_2 = \frac{1}{2} m v^2 = \frac{1}{2} \times 5 \times (10.19)^2$

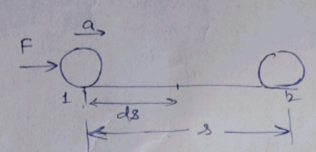
$KE_2 = 259.59 \text{ N-m}$

Change in $K.E = K.E_2 - K.E_1 = 259.59 - 0$
$= 259.59 \text{ N-m}$

W/D by net force on the body is equal to the change in K.E of the body.

Work Energy Principle:—

Consider a force F acting on an object which may displace from position 1 to position 2 over the course of action covering distance 's'.



For an elementary distance ds travelled by the objected in time 'dt' the work done by the force,

$$dW = F \cdot ds \quad —(1)$$

Newton's II nd Law of motion

$$F = m \cdot a$$

$$a = \frac{dv}{dt} = \frac{dv}{ds} \times \frac{ds}{dt} = v \cdot \frac{dv}{ds}$$

$$dW = \left(m \times v \times \frac{dv}{ds}\right) \times ds$$

$$dW = m v \, dv \quad —(2)$$

Integrating the above

$$\int_1^2 dW = m \int_1^2 v \, dv$$

$$W_{1-2} = m \cdot \left[\frac{v^2}{2}\right]_1^2$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$W_{1-2} = (K \cdot E)_2 - (K \cdot E)_1 \quad —(3)$$

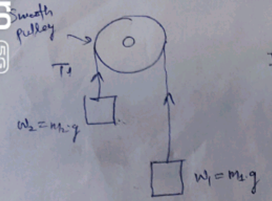
The ~~[illegible]~~ on the object equals to the change in Kinetic energy of the object.

The eqn 3 represents the well-known 'Work-energy principle' in mechanics

Motion of connected Bodies:—

(1.) Motion of two bodies connected by a string passing over a smooth pulley:—

Consider two bodies of masses m_1 and m_2 connected at the ends of a light inextensible string that passes over a smooth pulley. If $m_1 > m_2$.



If $m_1 > m_2$, then the mass m_1 moves downwards and the m_2 move upwards.

Downward motion of mass m_1:

$$m_1 g - T = m_1 a \quad \text{— (I)}$$

Upward motion of mass m_2

$$T - m_2 g = m_2 a \quad \text{— (II)}$$

By eqn (I) & (II) —

$$\boxed{a = \frac{m_1 - m_2}{m_1 + m_2} \cdot g}$$

$$\boxed{T = \frac{2\, m_1 \cdot m_2}{(m_1 + m_2)} \cdot g}$$

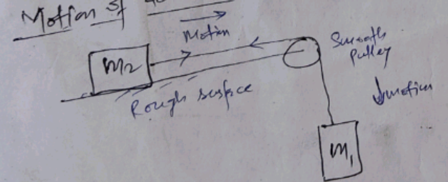
Press^r on (or resultant) the pulley equal to sum of tension T_1 and T_2 induced in the string lying on two sides of the pulley & since $T_1 = T_2 = T$ act along the same direction i.e. θ b/w is zero

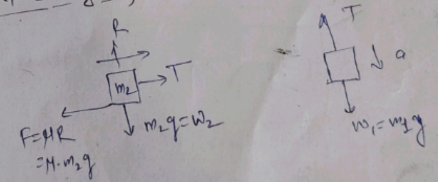
$$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta} \Rightarrow \sqrt{T^2 + T^2 + 2 \times T \times T \cos 0}$$

$$R = 2T$$

$$\boxed{R = \frac{4\, m_1 \cdot m_2}{m_1 + m_2} \cdot g}$$

Motion of two bodies connected at the edge of a horizontal surface:—





Horizontal motion of mass m_2: $T - \mu \cdot m_2 g = m_2 a$ — (1)

Downward motion of mass m_1 = $m_1 g - T = m_1 a$ — (2)

By eq^n (I) & (II) —

$$\boxed{a = \frac{m_1 - \mu m_2}{(m_1 + m_2)} = \frac{W_1 - \mu W_2}{W_1 + W_2} g}$$

$$\boxed{T = \frac{W_1 \cdot W_2 (1+\mu)}{W_1 + W_2}}$$

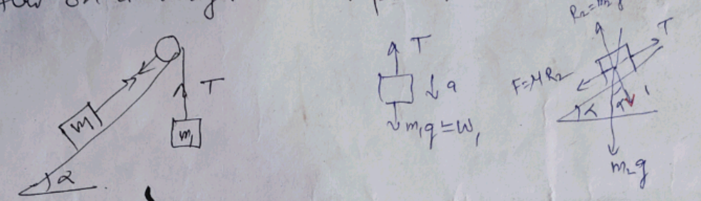
Press^r on the pulley $T_1 = T_2 = T$ they acts $\perp^r$ to each other i.e

$$\theta = 90^\circ$$

$$R = \sqrt{T^2 + T^2 + 2 \times T \times T \cdot \cos 90^\circ}$$

$$\boxed{R = \sqrt{2}\, T}$$

(3) Motion of two bodies connected by a string: one hangs free and the other on a rough inclined plane: —



Let mass m_1 move downwards,

$$m_1 g - T = m_1 a \quad \text{—— (I)}$$

$$T - m_2 g \sin\alpha - \mu m_2 g \cos\alpha = m_2 a \quad \text{—— (II)}$$

By eq^n (I) & (II): —

$$\boxed{a = \frac{W_1 - W_2 \sin\alpha - \mu W_2 \cos\alpha}{(W_1 + W_2)} \times g = \frac{m_1 g - m_2 g \sin\alpha - \mu m_2 g \cos\alpha}{(m_1 + m_2)}}$$

$$\boxed{T = m_1 \left[\frac{m_2 g (1 + \sin\alpha + \mu \cos\alpha)}{m_1 + m_2}\right] = \frac{W_1 \cdot W_2 (1 + \sin\alpha + \mu \cos\alpha)}{(W_1 + W_2)}}$$

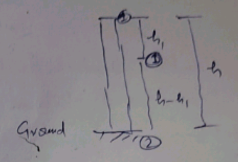
For smooth surface $\mu = 0$,

$$\boxed{a = \frac{W_1 - W_2 \sin\alpha}{W_1 + W_2} \cdot g} \quad \& \quad \boxed{T = \frac{W_1 \cdot W_2 (1 + \sin\alpha)}{(W_1 + W_2)}}$$

Press^r on pulley $= \sqrt{2(1 + \sin\alpha)}$

Conservation of Mechanical Energy:— The energy can neither be created nor destroyed through it can be transformed from one form to another.

Consider a body of mass m resting on the top of a tower of height 'h'. At this position,



K.E of the body $= \frac{1}{2} m v^2 = 0 \quad (\because v=0)$

P.E of the body with respect to ground $= m.g.h$

Total energy of the body at the top of tower is $= K.E + P.E$

$= 0 + m.g.h = m.g.h$ — (1)

Energy at position 1:—

Let the body fall to position 1 which is at distance h_1 from the top of the tower. The velocity of the body at this location can be worked out from the Kinematic eqn $v^2 - u^2 = 2as$.

$v_1^2 = 0^2 + 2.g\,h_1$

$v_1^2 = 2g\,h_1$

$\therefore$ K.E of the body at position 1 $= \frac{1}{2} m.v^2 = \frac{1}{2} m \times 2 \times g \times h_1$

$= mgh_1$

P.E of the body at position 1 $= mg(h-h_1)$

$\therefore$ Total energy of the body at position 1 $= mgh_1 + mg(h-h_1)$

$= mgh$ — (2)

Energy at position 2:—

Finally when the body has the complete fall from the top of tower to ground level. (position 2) we have —

$v_2^2 = u^2 + 2gh \Rightarrow v_2^2 = 2gh$

K.E at ground level $= \frac{1}{2} m v^2 = \frac{1}{2} m \times 2gh = mgh$

P.E at ground level $= mg \times 0 = 0 \quad (h=0)$ -

$\therefore$ Total energy of the body at ground level $= mgh + 0$

$= mgh$ — (3)

The above analysis does illustrate that the sum of K.E and P.E of the body remain constant under the action of gravity force. (a conservative force). Thus, through the K.E and P.E individually change throughout the motion, their sum always remain constant.

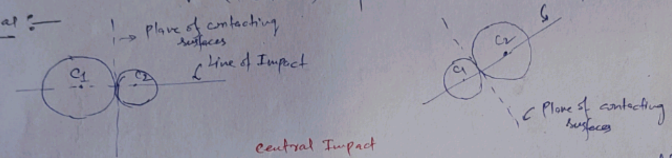
Inclined or oblique Impact:—

Impact:— The phenomenon of collision between two bodies which occures in a very short interval of time and during which the bodies exert relatively large forces on each other is called an impact.

Types of Impact:— when the two bodies collides with one another, the resulting impact may be—

(i) Central and eccentric Impact
(ii) Direct and oblique (inclined) Impact

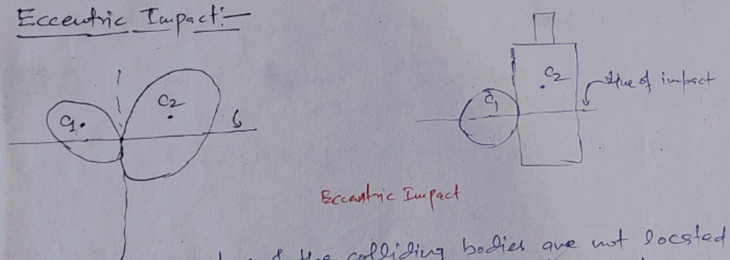
Central:—



Central Impact

⇒ The impact is called central when the mass centre of the colliding bodies are located on the line of impact.
The mass centres c_1 & c_2 of the colliding balls lie on the line of impact

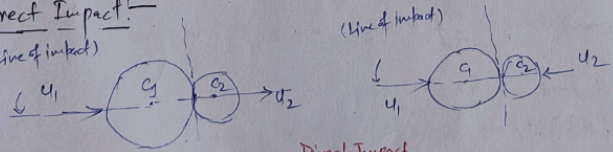
Eccentric Impact:—



Eccentric Impact

when the mass centres of the colliding bodies are not located on the line of impact, the impact referred to as eccentric impact.

Direct Impact:—

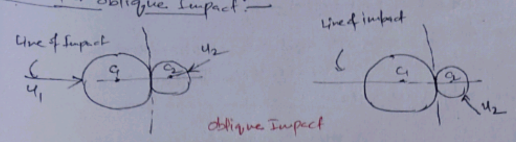


Direct Impact

$u_1 > u_2$

The impact is said to be direct if before impact the bodies are moving along the line of impact i.e. the motion of the colliding bodies directed along the line of impact.

Indirect or oblique Impact:—



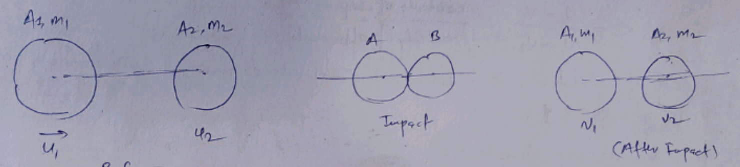
Oblique Impact

The impact is indirect or oblique if the motion of one or both the colliding bodies, before impact is not directed along the line of impact.

Elastic & Inelastic Impact:—

- The impact is elastic if the body rebounds after impact. Greater the elasticity of the body, greater will be rebound.
- The impact is inelastic if the body does not rebound at all.
- The property of bodies which leads to rebound after impact is called elasticity.

Conservation of momentum:—



(Before Impact) (After Impact)

Consider two bodies A & B of mass m_1 and m_2 respectively. Let these bodies be moving with respective velocities of u_1 & u_2 before impact & v_1 and v_2 after impact.

During collision, impulse $F \times t$ exerted by body A on body B. This impulse on body B is measured by the change in momentum.

Impulse on body B = change in momentum of body B

$$F \times t = m_2 v_2 - m_2 u_2 \quad \text{— (1)}$$

Acc^n to Newton third law of motion,

$$-F \times t = m_1 v_1 - m_1 u_1$$

$$F \times t = m_1 u_1 - m_1 v_1 \quad \text{— (2)}$$

From eq^n (1) & (2) —

$$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2$$

$$\boxed{m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2}$$

If the two bodies moving with velocity u_1 & u_2 before impact get coupled after the collision and move together in the same direction with velocity v, then

$$\boxed{m_1 u_1 + m_2 u_2 = (m_1 + m_2) v}$$

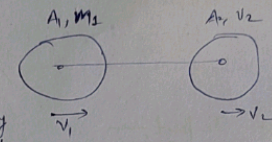
when the colliding bodies are inelastic, eg impact b/w two putty ball $(e=0)$

$$\boxed{\Delta E = \frac{m_1 m_2}{2(m_1+m_2)}(u_1-u_2)^2}$$

Newton's Law of collision: Coefficient of Restitution:—

Consider two bodies A and B of mass m_1 and m_2 respectively. Let these bodies be moving with respective velocities u_1 & u_2 before impact. The impact will take place only if $u_1 > u_2$

$\therefore$ velocity of approach $= (u_1 - u_2)$



After a short period of contact, the bodies will separate and will start moving with velocities V_1 and V_2 respectively. The separation will occur only if $V_2 > V_1$

velocity of separation $= V_2 - V_1$

New's Law of collision for elastic bodies states—

" when two moving bodies collide with each other, their velocity of separation bears constant ratio of their velocity of approach."

$$(V_2 - V_1) = e \cdot (u_1 - u_2)$$

$$e = \frac{V_2 - V_1}{u_1 - u_2} = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

where coefficient of restitution $= e$, which indicates the energy loss during an impact. $0 < e < 1$.

- If $e=0$, the bodies are inelastic
- If $e = 1$, the bodies are perfectly elastic.

The value of coeff. of restitution depend not only on the material but it also depends on the shape and size of the body.

Case <I> :— when the colliding bodies are inelastic, e.g. impact b/w two putty ball $(e = 0)$

$$\boxed{\Delta E = \frac{m_1 m_2}{2(m_1 + m_2)} (u_1 - u_2)^2}$$

Case <II> :— when the colliding bodies are perfectly elastic, e.g. impact b/w two hardened and polished steel balls $(e = 1)$

$$\boxed{\Delta E = 0}$$

Perfectly elastic impact :—

<i> momentum is conserved i.e.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

<ii> K.E is conserved i.e. $\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

<iii> The bodies separate after the impact.

<iv> $e = 1$ (The coefficient of restitution is unity).

<v> There is no permanent deformation in the bodies during the impact.

Partially Elastic impact :—

<i> Momentum is conserved i.e.—

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

(2) There is loss of K.E during impact.

<3> The two bodies separate after the impact.

<4> $0 < e < 1$

<5> Some permanent deformation remains in the body.

Plastic Impact :—

<1> The two bodies moves together with common velocity after the impact

<2> momentum is conserved.

$$m_1 u_1 + m_2 u_2 = (m_1 + m_2) \cdot v$$

<3> There is loss in K.E after the impact.

<4> $e = 0$

Impact with a fixed surface :—

The situation occurs when a body of certain mass ~~and~~ moving with a certain velocity strikes a wall.

<i> Momentum is not conserved.

<ii> Loss ~~of~~ K.E

<iii> $e = \frac{-v}{u}$ where u and v are velocities of the body before and after the impact.

a particle ... is known

Loss of Kinetic energy during Impact:—

Consider two bodies A and B which experience direct impact let

m_1, u_1, v_1 = mass, initial velocity and final velocity for body A

m_2, u_2, v_2 = mass, initial velocity & final velocity for body B.

Then, the K.E of two masses before impact $= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$

K.E of the two masses after impact $= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$

Loss of K.E during impact $= \frac{1}{2}\left[(m_1u_1^2 + m_2u_2^2) - (m_1v_1^2 + m_2v_2^2)\right]$

$$= \frac{1}{2(m_1+m_2)}\left[(m_1+m_2)(m_1u_1^2+m_2u_2^2) - (m_1+m_2)(m_1v_1^2+m_2v_2^2)\right]$$

$$= \frac{1}{2(m_1+m_2)}\left[\{m_1^2u_1^2 + m_2u_2^2 + m_1m_2(u_1^2+u_2^2)\} - \{m_1^2v_1^2 + m_2^2v_2^2 + m_1m_2(v_1^2+v_2^2)\}\right]$$

$\because m_1^2u_1^2 + m_2^2u_2^2 = (m_1u_1 + m_2u_2)^2 - 2m_1 \cdot m_2 \cdot u_1 \cdot u_2$

and $m_1m_2(u_1^2+u_2^2) = m_1 \cdot m_2(u_1-u_2)^2 + 2m_1m_2u_1 \cdot u_2$

$\therefore m_1^2u_1^2 + m_2^2u_2^2 + m_1m_2(u_1^2+u_2^2) = (m_1u_1+m_2u_2)^2 - 2m_1 \cdot m_2 u_1 u_2 + m_1 \cdot m_2(u_1-u_2)^2 + 2m_1m_2u_1u_2$

$$= (m_1u_1 + m_2u_2)^2 + m_1 \cdot m_2(u_1-u_2)^2$$

Similarly $\Rightarrow m_1^2v_1^2 + m_2^2v_2^2 + m_1m_2(v_1^2+v_2^2) = (m_1v_1+m_2v_2)^2 + m_1 \cdot m_2(v_1-v_2)^2$

$\therefore$ Loss K.E

$$\Delta E = \frac{1}{2(m_1+m_2)}\left[(m_1 \cdot u_1 + m_2u_2)^2 + m_1m_2(u_1-u_2)^2 - (m_1v_1+m_2v_2)^2 - m_1m_2(v_1-v_2)^2\right]$$

From, the Law of conservation of momentum,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

$(m_1u_1 + m_2u_2)^2 = (m_1v_1 + m_2v_2)^2$

$$\Delta E = \frac{1}{2(m_1+m_2)}\left[m_1m_2(u_1-u_2)^2 - m_1 \cdot m_2(v_1-v_2)^2\right]$$

$\because e = \frac{v_2 - v_1}{u_1 - u_2}$, $v_2 - v_1 = e(u_1-u_2)$

$$\Delta E = \frac{1}{2(m_1+m_2)}\left[m_1 \cdot m_2(u_1-u_2)^2 - m_1 \cdot m_2(v_1-v_2)^2\right]$$

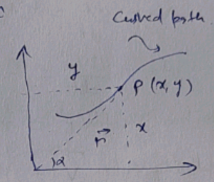
$$\boxed{\Delta E = \frac{m_1 \cdot m_2}{2(m_1+m_2)}(u_1-u_2)^2(1-e^2)}$$

Curvilinear Motion:— The motion of a particle along a curved path than a straight line is known as curvilinear motion.

Exp:— ① an automobile vehicle negotiating a turn on the road.
② projectile motion of a bullet fired from a gun.
③ Motion of bob of pendulum oscillating in vertical plane.
④ motion of satellite around the earth.

Rectangular-cartesian x, y coordinates:—

The position of the particle on the curved path at any instant is defined by the position vector $\vec{r} = x\hat{i} + y\hat{j}$
where $\hat{i}, \hat{j}$ are unit vector



Magnitude $r = |\vec{r}| = \sqrt{x^2+y^2}$

Velocity vector $\vec{V} = \frac{d}{dt}(\vec{r}) = \frac{d}{dt}(x\hat{i}+y\hat{j})$

Since $\hat{i}$ & $\hat{j}$ are constant,

$$\vec{V} = \hat{i}\frac{dx}{dt} + \hat{j}\frac{dy}{dt}$$

$$\vec{V} = V_x\hat{i} + V_y\hat{j}$$

Resultant velocity $\boxed{V = \sqrt{V_x^2 + V_y^2}}$

The direction of velocity is tangential to the path of motion of the particle. If α is the angle made by the resultant with x-axis, then

$$\boxed{\tan\alpha = \frac{V_y}{V_x}}, \quad \alpha = \tan^{-1}\left(\frac{V_y}{V_x}\right)$$

Acceleration $\vec{a} = \frac{d\vec{V}}{dt} = \frac{d}{dt}(V_x\hat{i}+V_y\hat{j})$

$$= \frac{d}{dt}\left(\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\right)$$

$$= \frac{d^2x}{dt^2}\hat{i} + \frac{d^2y}{dt^2}\hat{j}$$

$$\vec{a} = a_x\hat{i} + a_y\hat{j}$$

Resultant acceleration $a = \sqrt{a_x^2 + a_y^2}$

If β is the angle made by resultant acceleration with x-axis then

$$\boxed{\tan\beta = \frac{a_y}{a_x}} \qquad \boxed{\beta = \tan^{-1}\left(\frac{a_y}{a_x}\right)}$$

For a curvilinear motion in space,

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$$
$$\vec{V} = V_x\hat{i} + V_y\hat{j} + V_z k$$
$$\vec{a} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$$

Q. The motion of a particle is defined by the relations: $x = t^2+3t$ & $y = t^3-8t^2+3$ whose x and y are in meter. and t is in seconds.
(a) Write the eq^n defining the motion of the particle in vectorial form
(b) Calculate the velocity & acceleration of the particle at $t=2$ second

Sol^n:— $x = t^2+3t$ $\quad$ $y = t^3-8t^2+3$

$V_x = \frac{dx}{dt} = 2t+3$ $\quad$ $V_y = 3t^2-16t$

$a_x = \frac{dV_x}{dt} = 2$ $\quad$ $a_y = \frac{dV_y}{dt} = 6t-16$

Position Vector $\vec{r} = x\hat{i} + y\hat{j} \Rightarrow (t^2+3t)\hat{i} + (t^3-8t^2+3)\hat{j}$

velocity Vector $\vec{V} = V_x\hat{i} + V_y\hat{j} \Rightarrow (2t+3)\hat{i} + (3t^2-16t)\hat{j}$

Acceleration $\vec{a} = a_x\hat{i} + a_y\hat{j} = 2\hat{i} + (6t-16)\hat{j}$

(b) At $t=2$ seconds.

$\vec{V} = (2\times2+3)\hat{i} + (3\times2^2-16\times2)\hat{j}$

$V = 7\hat{i} - 20\hat{j}$ $\quad$ $V_x = 7$ m/s, $V_y = -20$ m/s

$V = \sqrt{V_x^2+V_y^2} \Rightarrow \sqrt{7^2+(20)^2} = 21.19$ m/s

$\tan\alpha = \frac{V_y}{V_x} = \frac{20}{7} \Rightarrow \alpha = 70.71°$

$|\vec{a}| = 2\hat{i} + (6t-16)\hat{j}$

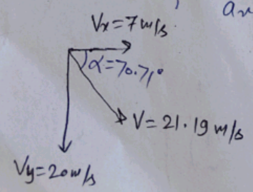
At $t=2$ sec $\quad \vec{a} = 2\hat{i} + (6\times2-16)\hat{j} \Rightarrow \vec{a} = 2\hat{i} - 4\hat{j}$

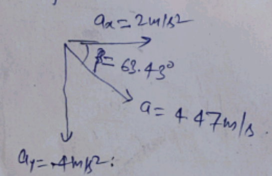
$a_x = 2$ m/s², $a_y = -4$ m/s²

$a = \sqrt{a_x^2+a_y^2} \Rightarrow \sqrt{2^2+(-4)^2}$

$a = 4.47$ m/s²

$\tan\beta = \frac{a_y}{a_x} = \frac{4}{2} \Rightarrow \beta = 63.43°$

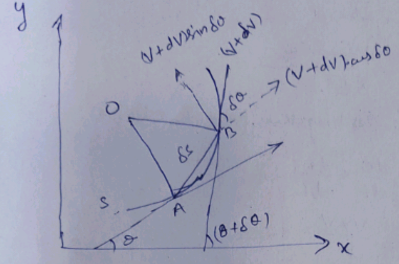




... mass 500 Tonne moving at 5 km/hr collides with the back of another ... same direction at 1.5 km/hr

Tangential and Normal coordinates :— In this system, the velocity and accelaration of a moving particle are expressed in tangential (t) & normal (n) components.

Consider a particle that moves along a curved path from point A to point B. and transverses an infinitely small distance δs in small interval of time δt.



The velocity of the particle at point A in the tangential direction is —

$$V_t = \frac{\text{distance moved along the tangential direction}}{\text{time interval}}$$

$$= \lim_{\delta t \to 0} \frac{\delta s \cdot \cos\delta\theta}{\delta t}$$

$$= \lim_{\delta t \to 0} \frac{\delta s}{\delta t} \cdot \cos\delta\theta$$

As $\delta\theta$ is small, $\cos\delta\theta \to 1$

$$V_t = \lim_{\delta t \to 0} \frac{\delta s}{\delta t} = \frac{ds}{dt}$$

The velocity of the particle in the normal direction

$$V_n = \frac{\text{Distance moved along the tangential direction}}{\text{Time interval}}$$

$$= \lim_{\delta t \to 0} \frac{\delta s \cdot \sin\delta\theta}{\delta t}$$

As $\delta\theta$ is small, $\delta\theta \to \delta\theta$ Neglecting the product of $\delta s \cdot \delta\theta$

$$V_n = 0$$

Tangential accelaration $a_t = \frac{\text{change in tangential velocity}}{\text{Time interval}}$

$$= \lim_{\delta t \to 0} \frac{(V+\delta V)\cdot\cos\theta - V}{\delta t}$$

when δt is very small, the arc AB will be very small & $\delta\theta$ will be $\delta\theta$.
$\cos\delta\theta = 1$

Wagon A of mass 100 Tonne moving at 8 Km/hr collides with the back of another wagon B of mass 40 tonne and moving in the same direction at 1.5 km/h.

$$a_t = \frac{dV}{dt}$$

Normal acceleration $a_n = \dfrac{\text{change in normal velocity}}{\text{Time interval}}$

$$= \lim_{\delta t \to 0} \frac{(V+\delta V)\sin\delta\theta - 0}{\delta t}$$

For small $d\theta$, $\sin\delta\theta = \delta\theta$, Neglecting the product $(\delta V \cdot \delta\theta)$

$$a_n = V\cdot\frac{d\theta}{dt} = V\cdot\frac{d\theta}{ds}\times\frac{ds}{dt}$$

$$= \frac{V\cdot ds/dt}{ds/d\theta} = \frac{V\times V}{\rho}$$

$$a_n = \frac{V^2}{\rho}$$, where $\rho = \dfrac{ds}{d\theta}$ is the radius of curvature of the path.

when the body moves in a circular orbit of radius r, then

$$a_n = \frac{V^2}{r}$$

Resultant acceleration $a = \sqrt{a_t^2 + a_n^2}$

$\tan\phi = \dfrac{a_t}{a_n}$, where ϕ, resultant acceleration angle

Q. A motorist is driving at 80 km/hr on the curved portion of a highway of 400m radius. He suddenly applies the brakes and that causes the speed to decrease to 45 km/hr at a constant rate in 8 sec. Determine the tangential & normal components of acceleration immediately after the application of brakes and 4 sec. later.

Soln:- Initial velocity $u = 80$ km/hr $\Rightarrow 80\times\frac{5}{18}$ m/s $\Rightarrow 22.22$ m/s.

$V = 45\times\frac{5}{18} \Rightarrow V = 12.5$ m/s.

Tangential acceleration = rate of change of velocity

$$a_t = \frac{12.5 - 22.22}{8} \Rightarrow a_t = -1.215 \text{ m/s}^2$$

$$a_n = \frac{V^2}{r} \Rightarrow \frac{22.22^2}{400} \Rightarrow \quad a_n = 1.234 \text{ m/s}^2$$

Total acceleration $a = \sqrt{a_n^2 + a_t^2} \Rightarrow \sqrt{(1.234)^2 + (-1.215)^2} \Rightarrow \boxed{a = 1.732 \text{ m/sec}}$

(b) velocity of vehicle after 4 sec. (application of brakes)

$V = u + at \Rightarrow 22.22 + (1.215\times 4) \Rightarrow V = 17.36$ m/s²

$a_n = \dfrac{V^2}{r} = \dfrac{17.36^2}{400} \Rightarrow a_n = 0.753$ m/s², $a_t = -1.215$ m/s²

$a = \sqrt{a_n^2 + a_t^2}$ $\sqrt{(0.753)^2 + (-1.215)^2} \Rightarrow \boxed{a = 1.429 \text{ m/s}^2}$

Q. Wagon A of mass 100 Tonne moving at 5 Km/hr collides with the back of another wagon B of mass 40 tonne and moving in the same direction at 1.5 Km/h. After impact, the wagon B sets moving with a velocity of 7.5 km/h. Determine the velocity of wagon A after the impact and impulse b/w the two wagons.

Soln:— From Law of conservation of momentum

$$m_1 u_1 + m_2 u_2 = m_1 V_1 + m_2 V_2$$ (Let suffix 1 for wagon A, Suffix 2 for wagon B)

$$100 \times 5 + 40 \times 1.5 = 100 \times V_1 + 40 \times 7.5$$

$$\boxed{V_1 = 2.6 \text{ km/h}}$$

$\Rightarrow$ Impulse on each wagon equals to change in its momentum.

$m = 100$ tonne $= 100 \times 10^3$ kg

$u_1 = 5 \times \frac{5}{18} = 1.39$ m/s

$V_1 = 2.6 \times \frac{5}{18} = 0.72$ m/s

Impulse = mass × change in velocity

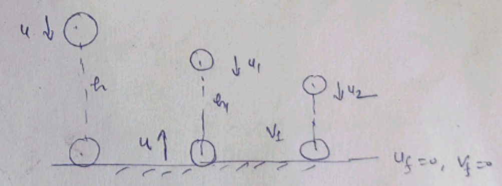
$= 100 \times 10^3 \times (1.39 - 0.72)$

$= 67000$ N-sec Ans.

Q. A ball is dropped from a height of 10 m on a smooth floor and it rebounds to a height of 7 m. Determine the coefficient of restitution b/w the ball and floor and expected height of second rebounce.

Soln:— $u = \sqrt{2 \cdot g \cdot h}$ ↓

and $v = \sqrt{2 \cdot g \cdot h_1}$



The velocity of floor is zero before and after every impact i.e.,

$u_f = 0$, $v_f = 0$

$$e = \frac{v_f - v}{u - u_f} = \frac{-v}{u}$$

Since u & v are in opposite direction

$$e = \frac{v}{u} = \frac{\sqrt{2g h_1}}{\sqrt{2g h}} \Rightarrow e = \sqrt{\frac{h_1}{h}} \Rightarrow e = \sqrt{\frac{7}{10}} \Rightarrow e = 0.837$$

$\therefore \frac{h_1}{h} = e^2$ $\quad h_1 = e^2 h$

$h_2 = e^2 h_1$

Height at second rebounce $h_2 = 0.837^2 \times 10$

$= 4.9$ m

Instantaneous Centre:— The place ...

Relative Motion:— The motion relative to a set of axes which are moving is called the relative motion.

For example, the motion of a train A with respect to another moving train B is the relative of the ~~m~~ train A with respect to the train B.

Relative velocity: Working concept:—

Consider two trains A and B travelling on two straight and parallel tracks with the speed of 100 km and 80 km/hr respectively.

• When moving in the same direction:—

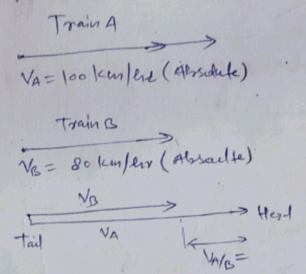
$V_A = 100$ km/hr

$V_B = 80$ km/hr

Relative Velocity $V_{A/B} = V_A - V_B$

$= 100 - 80$

$= 20$ km/hr



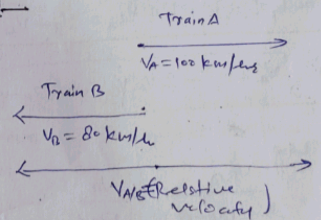
Relative velocity $V_{A/B}$ = Absolute velocity of A − Absolute velocity of B

• When moving in opposite direction:—

$V_{AB} = V_A - V_B$

$= [100 - (-80)]$

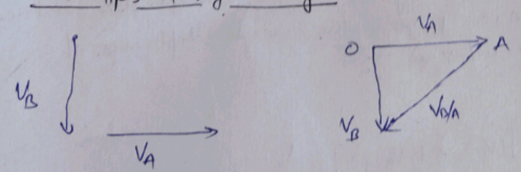
$= 180$ km/hr



Relative velocity of A w.r.t B = the vector difference of absolute velocities of A and B.

Determination of the relative velocity of A w.r.t B is essentially the determination of the vector difference of their velocities V_A and V_B.

Velocities at right angles:—



Relative velocity of B w.r.t A

$\vec{OA} + \vec{AB} = \vec{OB}$

$\vec{AB} = \vec{OB} - \vec{OA}$

$V_{B/A} = V_B - V_A$
(From A to B)

Relative velocity of A w.r.t B.

$\vec{OB} + \vec{BA} = \vec{OA}$

$\vec{BA} = \vec{OA} - \vec{OB}$

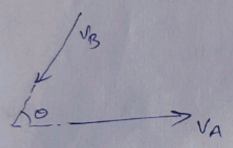
$V_{A/B} = V_A - V_B$
(From of B to A)

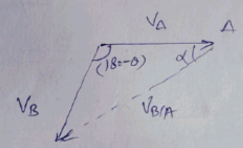
Magnitude of the relative velocity $= \sqrt{V_A^2 + V_B^2}$

Direction $\tan\theta = \frac{V_B}{V_A}$

$V_{A/B}$ and $V_{B/A}$ have the same magnitude but opposite direction.

<u>velocities at angle θ</u> :—





Relative velocity of B w.r.t A $V_{B/A} = V_B - V_A$

$$\boxed{V_{B/A} = \sqrt{V_A^2 + V_B^2 - 2 V_A \cdot V_B \cos(180-\theta)}}$$

using sine Law

$$\frac{V_{B/A}}{\sin(180-\theta)} = \frac{V_B}{\sin\alpha}$$

$$\Rightarrow \boxed{\sin\alpha = \frac{V_B \sin(180-\theta)}{V_{B/A}}}$$

Alternative Approach :— Problems in relative motion can also solved by expressing the absolute and relative velocities of object as vector and then following the simple concept that relative velocity of A w.r.t B is the vector difference of two velocity ~~vector~~ as.

(W)

$$V_{A/B} = V_A - V_B$$

$$[(V_{A/B})_x \hat{i} + (V_{A/B})_y \hat{j}] = [(V_A)_x \hat{i} + (V_A)_y \hat{j}] - [(V_B)_x \hat{i} + (V_B)_y \hat{j}]$$

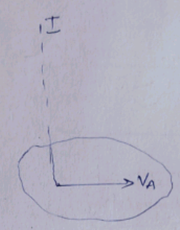
Instantaneous Centre:— The plane motion of all the particles constituting the body may then be considered as pure rotation about that point. Such a point is called the instantaneous centre or virtual centre of body.

The axis passing through this point and at right angles to the plane of motion is called instantaneous axis of rotation.

· The I.C changes every moment, and its locus is called centrode.

Let V_a and ω be the linear and angular velocities of a point A on a rigid body.

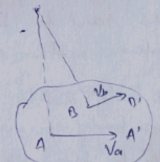
Since 'ω' and V_a are connected by the expression $V = \omega . r$, the instantaneous centre 'I' then lies at a distance $\frac{V_a}{\omega}$ along the perpendicular to direction of velocity V_a at point A



$$I\text{-}A = \frac{V_a}{\omega}$$

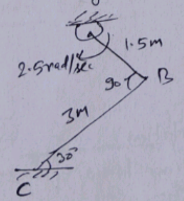
Let V_a & V_b be the linear velocities at points A and B on a rigid body.

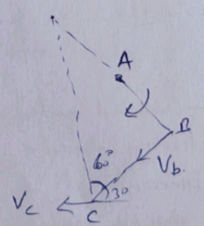
These velocities are directed along the direction on AA' and BB' respectively. The Instantaneous centre I is then the point of intersection of lines erected perpendicular to the direction of velocities at the given point.



Then $V_a = \omega \times I.A$ & $V_b = \omega \times IB$

Q. At the instant shown in Fig., the rod AB is rotating clockwise at 2.5 Rad/sec. If end C of the rod BC is free to move on a horizontal surface, make calculation for the angular velocity of rod BC and velocity of its end point C.





$V_b = \omega \times r$
$= 2.5 \times 1.5$
$= 3.75 \text{ m/s}$

$IC = \frac{BC}{\cancel{\sin}\cos 60} \Rightarrow \frac{3}{0.5} \Rightarrow IC = 6m$

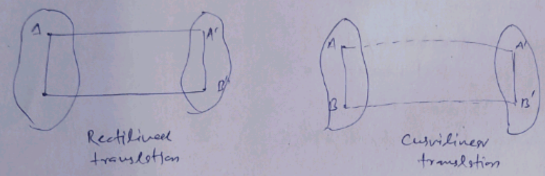
$IB = IC \sin 60° = 5.196 \text{ m}$

Angular velocity of rod BC, $\omega = \frac{V_b}{IB} = \frac{V_c}{I_c} \Rightarrow \omega = \frac{3.75}{5.196} \Rightarrow 0.722 \text{ rad/sec. (CW)}$

$V_c = \omega \times IC = 0.722 \times 6 = 4.33 \text{ m/s}$

General Plane Motion:— Generally a body undergoes the following three types of plane motions:

Translation:— The particles constituting the rigid body move in parallel planes and travel the same distance. During translation, the particles have the same velocity and acceleration and a straight line drawn on the moving body remains parallel to its original position at any time.



If the path traced by the particle during motion is a straight line, then the motion is said to be rectilinear translation. If the particle traces a curved path, the motion is called curvilinear translation.

Rotation:— The body rotates about a fixed point and all the particles constituting the body move in a circular path. The fixed point about which the body rotates is called the point of rotation and the axis passing through the point of rotation is called the axis of rotation. A point lying on the axis of rotation has zero velocity & zero acceleration.

General plane motion:— (combined motion of translation and rotation)
There exist certain situation where a body possesses both motions of translation and rotation simultaneously at a particular instant.
Examples of such a combined motion are—
- Motion of roller without slipping
- Motion of the wheel of a locomotive, train, truck and car etc

Beam:— A structural member subjected to external force or couples at right angles to the longitudinal axis is called a beam.

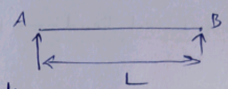
OR

Beam is structural member which is vertically loaded.

Types of Beam:—

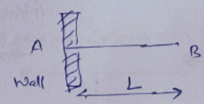
(1) Simple Supported Beam:—

A beam which is freely supported on the walls or columns at its both the ends is called as simply supported beam.



(2) Cantilever Beam:—

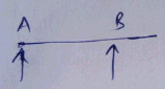
A beam fixed at one end and free at the other is called as a cantilever beam.



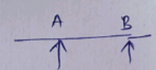
(3) Overhanging Beam:—

If the end portion of the beam extends beyond the support, it is called as an overhanging beam.

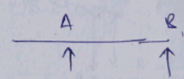
A Beam may be overhanging on one side or on both side as shown in figure.



overhanging on right side



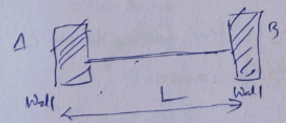
(overhanging on both side)



overhanging on left side

(4) Fixed Beam:—

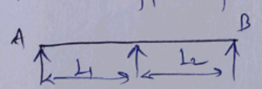
A beam whose both the ends are rigidly fixed in walls is called a fixed or constrained beam, built-in beam or an encastre beam.



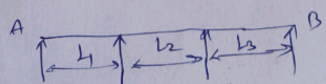
(5) Continues Beam:—

A beam which is supported on more than two supports ie (at least three supports) is called a continuous beam.

The end supports of a continuous beam may be simply supported or fixed.



Two span continuous beam

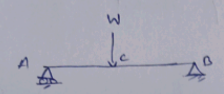


Three span continuous beam

Bending moment at a section of the beam is the

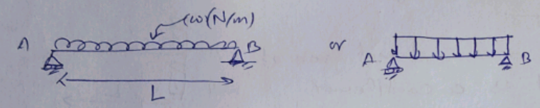
Types of Loads:—

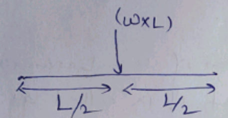
<1> **Concentrated Load:**— A concentrated load is one which is assumed to act at a point.



<2> **Uniformly distributed load:**— A uniformly distributed load is uniformly or evenly distributed over a part or over the entire length of beam.

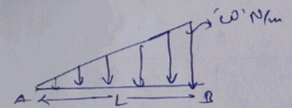
The rate of loading is expressed in N/m.

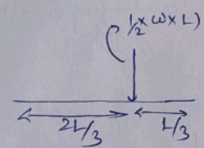




(converted to point load)

<3> **Uniformly Varying load:**— A load whose intensity of loading varies linearly or at constant rate along the length.

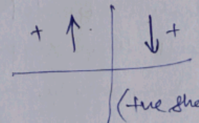




Shear force:— Shear force at a section of the beam is the force that is trying to shear off the section of beam.

Sign convention:—

(−ve sign)



(+ve shear)

Bending moment:— Bending moment at a section of the beam is the moment that tends to bend the beam.
It is obtained by algebric summation of moments of all the external force about the section acting either to the left or right portion of the section.

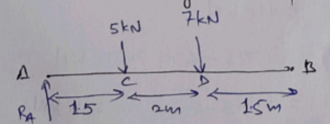
Sign convention:—

+ve (Sagging) (Concave)

−ve (Hogging) (Convex)

Note:— The curve for B.M in a portion of the beam is one degree higher than the curve of shear force.

Q A simply supported beam of span 5m carries two point loads of 5 kN and 7 kN as shown in Fig. Draw S.F.D & B.M.D showing the important values.



Sol:ⁿ $+\uparrow \Sigma F_y = 0$

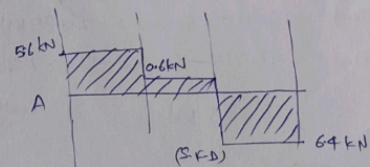
$R_A + R_B - 5 - 7 = 0$

$R_A + R_B = 12$ — (I) , $\boxed{R_A = 5.6\,kN}$

$\circlearrowright \Sigma M_A = 0$

$5 \times 1.5 + 7 \times 3.5 - R_B \times 5 = 0$

$\boxed{R_B = 6.4\,kN}$

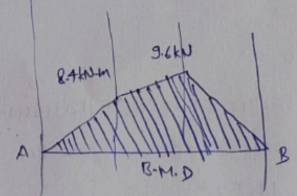


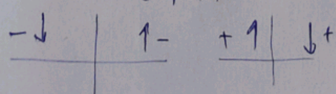
S.F. calculation

(i) S.F between A & C: $= 5.6\,kN$
or $5 + 7 - 6.4 = 5.6\,kN$

(ii) S.F b/w C & D $= 5.6 - 5 = 0.6\,kN$
or $7 - 6.4 = 0.6\,kN$

(iii) S.F b/w D & B $= 5.6 - 5 - 7 = -6.4\,kN$
or $-6.4\,kN$





(−ve shear) (+ve shear)

BM calculation:—

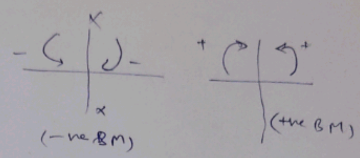
1) B M at point A = 0
or $\Rightarrow -5\times1.5 - 7\times3.5 + 6.4\times5 = 0$

2) B.M at point B = 0
or $\Rightarrow 5.6\times5 - 5\times3.5 - 7\times1.5 = 0$

3) B.M at point C = $5.6\times1.5 = 8.4$ kN-m
or $-7\times2 + 6.4\times3.5 \Rightarrow 8.4$ kN-m

4) B.M at point D = $5.6\times3.5 - 5\times2$
$= 9.6$ kN-m
or
$6.4\times1.5 = 9.6$ kN-m

Sign convention:—



Q.

$+\uparrow \Sigma F_y = 0$

$R_A + R_B - 6 - 3 - 2 - 5 = 0$

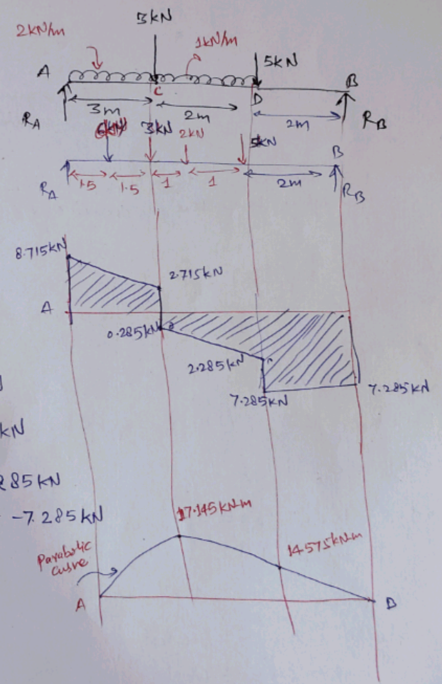
$R_A + R_B = 16$ —— (I)

$+\circlearrowright \Sigma M_A = 0$

$6\times1.5 + 3\times3 + 2\times4 + 5\times5 - R_B\times7 = 0$

$R_B = 7.285$ kN

$R_A = 8.7515$ kN



SF calculation:—

1) SF at A = 8.715 kN

2) SF at C (left) = $8.715 - 6 = 2.715$ kN

3) SF at C (right) = $8.715 - 6 - 3 = -0.285$ kN

4) SF at D (left) = $8.715 - 6 - 3 - 2 = -2.285$ kN

5) SF at D right = $8.715 - 6 - 3 - 2 - 5 = -7.285$ kN

6) SF at B = -7.285 kN

B. M. calculation:—

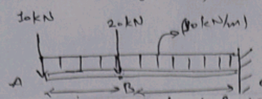
1) BM at A = 0

2) B.M at B = 0

3) B.M at C = $8.715\times3 - 6\times1.5 = 17.145$ kN-m

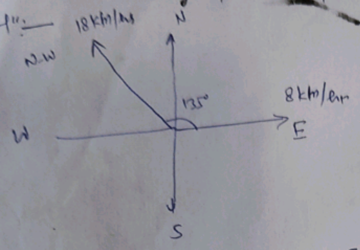
4) B M at D = $8.715\times5 - 6\times3.5 - 3\times2 - 2\times1$
$= 14.575$

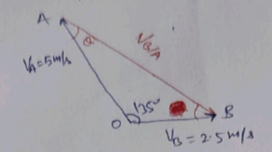
Construct shear force and bending moment diagram for the cantilever beam as shown in fig.



Q. Ship A is moving north-west at a speed of 18 km/hr and the ship B is moving east at a speed 9 km/hr. Find the magnitude and direction of the relative velocity of the ship B with respect to the ship A.

Solⁿ:—





Speed of the ship A, $V_A = 18 \times \frac{5}{18} = 5$ m/s

Speed of the ship B, $V_B = 9 \times \frac{5}{18} = 2.5$ m/s

$$\overline{V_{B/A}} = \overline{V_B} - \overline{V_A}$$

$$AB^2 = OA^2 + OB^2 - 2(OA)(OB)\cdot\cos 135^\circ \text{ (Cosine Law)}$$

$$V_{B/A}^2 = V_A^2 + V_B^2 - 2 \times V_A \cdot V_B \cdot \cos 135^\circ$$

$$= 5^2 + 2.5^2 - 2(5)(2.5) \times \left(-\frac{1}{\sqrt{2}}\right)$$

$$\boxed{V_{B/A} = 6.99 \text{ m/s}}$$

By Sine Law, $\frac{OB}{\sin\theta} = \frac{AB}{\sin\alpha}$

$$\frac{2.5}{\sin\theta} = \frac{6.99}{\sin 135}$$

$$\boxed{\theta = 14.65^\circ}$$

Alternative solution:—

$$\overline{V_A} = (-5\cos 45^\circ)\hat{i} + (5\sin 45^\circ)\hat{j}$$

$$\overline{V_A} = -3.535\hat{i} + 3.535\hat{j}$$

$$\overline{V_B} = 2.5\hat{i}$$

$$\overline{V_{B/A}} = \overline{V_B} - \overline{V_A}$$

$$= (2.5\hat{i}) - [-3.535\hat{i} + 3.535\hat{j}]$$

$$\overline{V_{B/A}} = 6.035\hat{i} - 3.535\hat{j}$$

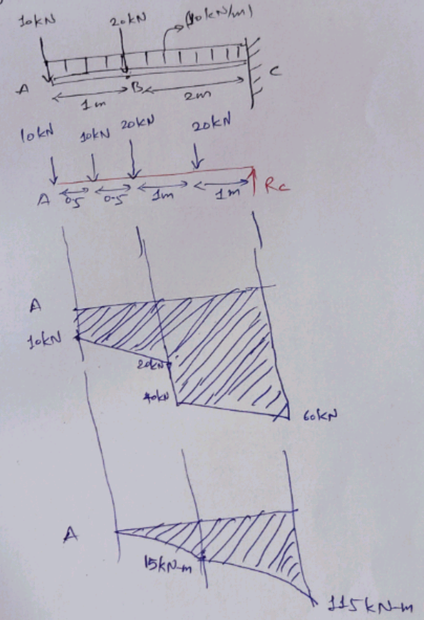
Magnitude $= \sqrt{(6.035)^2 + (3.535)^2}$

$$\overline{V_{B/A}} = 6.99 \text{ m/s}$$

$\tan\theta = \frac{V_B}{V_A} = \frac{3.5}{6}$ ... $= 30.35$ with horizontal

... $-30.35) = 14.65^\circ$ with the direction of ship A

Q. Construct shear force and bending moment diagram for the cantilever beam as shown in fig.



$\Sigma F_y = 0$

$\Rightarrow -10-10-20-20+R_C = 0$

$R_C = 60$ kN

SF calculation:—

SF at A $= -10$ kN

SF at B left $= -10-10$
$= -20$ kN

SF at B right $= -10-10-20$
$= -40$ kN

SF at Point C left $= -60$ kN

Bending moment calculation:—

B.M at point A $= 0$

BM at Point B $= -10 \times 1 - 10 \times 0.5$
$= 15$ kN-m

BM at Point C $= -10 \times 3 - 10 \times 2.5 - 20 \times 2 - 20 \times 1$
$= 115$ kN-m