UNIT-3 WAVE OPTICS

INTERFRENCE:- The phenomenon of addition or superposition of two light waves is called interference of light.

At some points the intensity is maximum while at other points the intensity is minimum.

Maximum intensity is called constructive interference.

Minimum intensity is called destructive interference.

When two light wave interfere we get alternate dark and bright ~~fringes~~ bands. These are called interference fringes.

Coherent Sources:- Two sources are said to be coherent if they emit light waves of same frequency, same amplitude and are in same phase with each other.

Temporal coherence: (Coherence in time)

Longitudinal coherence is known as temporal coherence. It is a ~~relation~~ measure of phase relation of wave reaching at a given points at two different times.

Spatial coherence (Coherence in space).

Transverse coherence or lateral coherence is known as spatial coherence. It is a measure of phase relationship between the waves reaching at two different points in space at same time.

Methods for producing interference pattern:-

It is divided into two parts.

I) **Division by wave front:-** Interference of light takes place between waves from two sources formed due to ~~two~~ single source.

Eg Interference by Young double slit.

II) **Division by Amplitude:-** Interference takes place between the waves from the ~~real~~ real source & virtual source.

Example- Interference by thin film.

Superposition:- (Combined effect of coherant waves)

When two or more waves cross at a point, the Displacement at ~~each point~~ that point is equal to the displacement of Individual wave

- When two or more coherant waves superimpose, the resultant Effect is brightness in central region and darkness at other region.

Fringes:- Alternate Bright and dark bands are fringes.

* The fringes pattern are obtained only when the interfering waves are coherant.
* Region of Brightness are darkness are also known as Maxima & Minima.

Monochromatic waves:- ~~Two or more waves maintain a constant phase difference~~

waves with single frequency & wavelength are Monochromatic waves.

* **Path of light:-**
* When light travel along straight line known as path of light
* Shortest path between any two points called Geometrical path. (GPL)
* **Optical path:-** wave travel μ times slower in a medium.

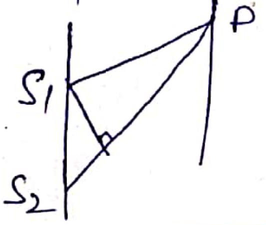
$$\text{OPath} = \mu(\text{GPL})$$

$$\boxed{\Delta = \mu L}$$

Path difference:- Difference between optical path of two rays travelling in different Direction known as optical path difference.

$$GPL = S_2P - S_1P$$

$$OPL = \mu(S_2P - S_1P)$$



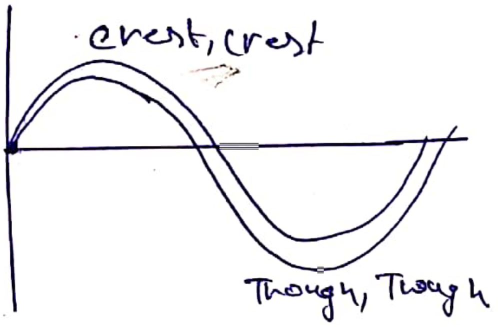
Path Difference & Phase Difference

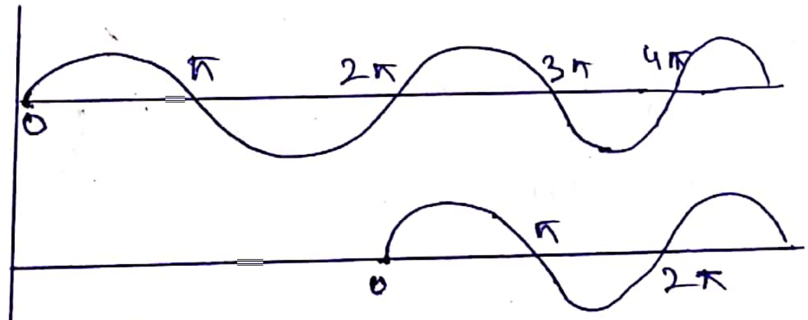
Path Difference:- Difference in the path travelled by the two waves.

Phase difference:-> Phase difference is Explained as the time gap where the wave Either falls behind or lead to another.

1) Constructive Interference:

occurs when the phase difference between the waves is an even Multiple of π $(2\pi, 4\pi, 6\pi ---)$

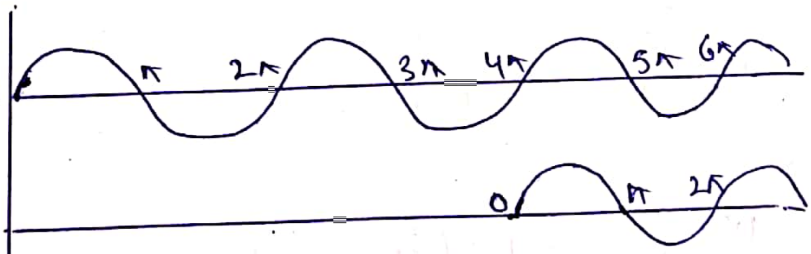




$2\pi - 0 = 2\pi$ $\quad$ $4\pi - 2\pi = 2\pi$

$3\pi - \pi = 2\pi$

or



$4\pi - 0 = 4\pi$

$5\pi - \pi = 4\pi$

1) Phase Difference ~~betw~~

$= 0, 2n\pi \Rightarrow (2\pi, 4\pi, 6\pi ---)$

11) Path Differen $= n\lambda$

or

$$\boxed{\text{Phase Difference} = \frac{2\pi}{\lambda} \times \text{Path Difference}}$$

(because $2\pi = \lambda$

$\pi = \lambda/2$

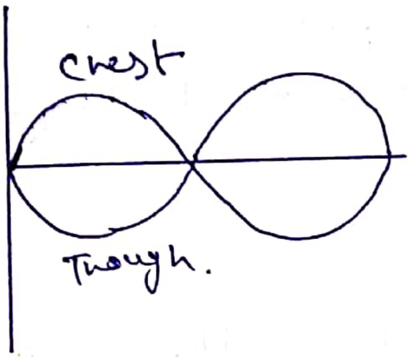
$\Rightarrow 2n\pi = \cancel{2 \times n \times \lambda}$

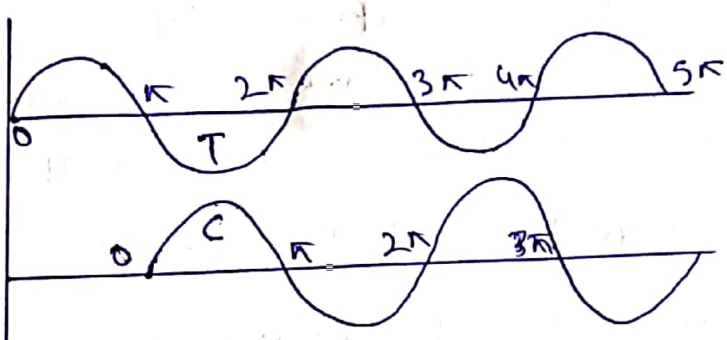
$= 2 \times n \times \frac{\lambda}{2} = \cancel{n\lambda}$

$= n\lambda$)

Destructive Interference:-

occurs when two waves cancel the effect of each other.
Phase difference between waves is an ~~even~~ odd multiply of π.





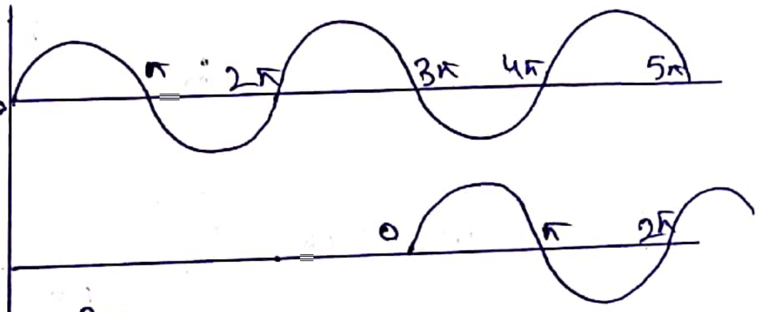
$\pi - 0 = \pi$

$2\pi - \pi = \pi$

$3\pi - 2\pi = \pi$

$4\pi - 3\pi = \pi$

odd
$(\pi, 3\pi, 5\pi ----)$



$3\pi - 0 = 3\pi$

$4\pi - \pi = 3\pi$

$5\pi - 2\pi = 3\pi$

i) Phase Difference $= (2n-1)\pi$ (odd)

ii) Path Difference $= (2n-1)\frac{\lambda}{2}$ (using $\pi = \lambda/2$)

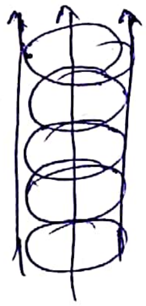
Conditions for Sustained Interference:-

I) Two sources must be monochromatic ie they must emit light of same wavelength or frequency.

II) Two sources must have either no phase difference or the phase difference must remain unchanged with time.

Huygen's Principle:-

All points on primary wavefront are considered to be center of disturbance and sends out secondary waves in all the directions which travels through space with same velocity in an isotropic medium.



Ratio of Intensity of light at Maxima & Minima

$$\frac{I_{max}}{I_{min}} = \cancel{\frac{(a_1^2+a_2^2)}{}} \frac{(a_1+a_2)^2}{(a_1-a_2)^2}$$

Ratio of Intensity of light due to two Sources:

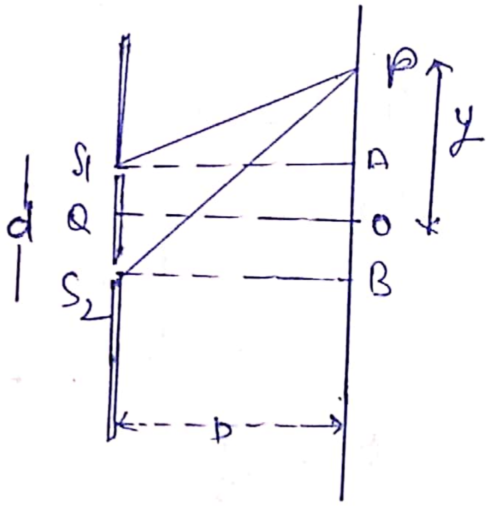
$$\frac{w_1}{w_2} = \frac{I_1}{I_2}$$ ($w_1, w_2 \rightarrow$ width of two slits S_1 & S_2)

$$\frac{I_1}{I_2} = \frac{a_1^2}{a_2^2}$$ ($a_1, a_2 \rightarrow$ Amplitude)

$$\Rightarrow \boxed{\frac{w_1}{w_2} = \frac{a_1^2}{a_2^2}}$$

Fringe width

$$\boxed{\beta = \frac{D\lambda}{d}}$$



D - Distance between slit & screen

d - Distance bet source S_1 & S_2

Position of bright fringes:-

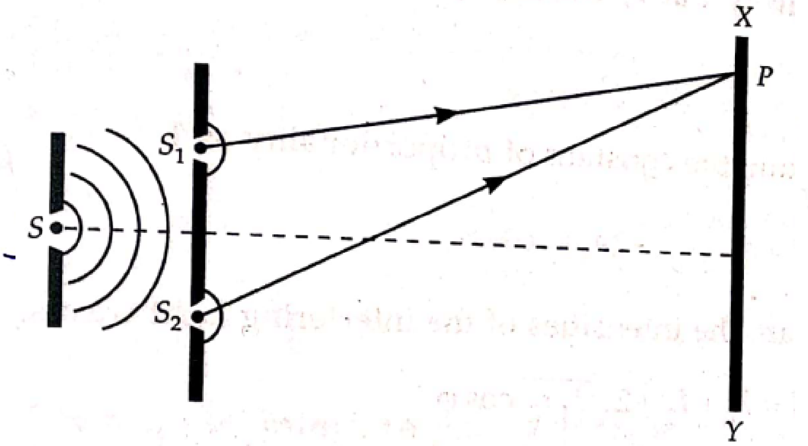
$$\boxed{y = \frac{nD\lambda}{d}}$$

$S_1P - S_2P$ = Path Difference

position of dark fringes:-

$$\boxed{y = \frac{(2n+1)D\lambda}{d}}$$

Conditions for Constructive & Destructive Interference:



Consider a monochromatic source of light S emitting waves of wavelength λ. S_1 & S_2 are two similar slits.

Let the phase difference between waves be φ

$$\varphi = \frac{2\pi}{\lambda} \times \text{Path difference}$$

$$= \frac{2\pi}{\lambda} \times (S_2P - S_1P)$$

Let y_1 & y_2 be the displacement of two waves

$$y_1 = a_1 \sin \omega t \quad \text{--- (1)}$$

$$y_2 = a_2 \sin(\omega t + \varphi) \quad \text{--- (2)}$$

Hence Resultant Displacement is

$$Y = y_1 + y_2$$

$$= a_1 \sin \omega t + a_2 \sin(\omega t + \varphi)$$

$$= a_1 \sin \omega t + a_2 \sin \omega t \cos \varphi + a_2 \cos \omega t \sin \varphi$$

$$= \sin \omega t (a_1 + a_2 \cos \varphi) + a_2 \cos \omega t \sin \varphi$$

Let $a_1 + a_2 \cos \varphi = R\cos\theta \quad \text{--- (3)}$

$a_2 \sin \varphi = R\sin\theta \quad \text{--- (4)}$

Hence $Y = \sin \omega t \, R\cos\theta + \cos \omega t \, R\sin\theta = R\sin(\omega t + \theta)$

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Squaring and Adding (3) & (4)

$$R^2(\cos^2\theta + \sin^2\theta) = (a_1 + a_2\cos\varphi)^2 + (a_2\sin\varphi)^2$$

$$\Rightarrow \quad = a_1^2 + a_2^2\cos^2\varphi + 2a_1a_2\cos\varphi + a_2^2\sin^2\varphi$$

$$\Rightarrow \boxed{R^2 = a_1^2 + a_2^2 + 2a_1a_2\cos\varphi}$$

But Intensity $I = R^2$

$$\Rightarrow \quad I = a_1^2 + a_2^2 + 2a_1a_2\cos\varphi \quad -(5)$$

Now if I_1 & I_2 are the intensities of interfering light

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\varphi$$

Suppose $a_1 = a_2 = a$

$$I = a^2 + a^2 + 2a^2\cos\varphi$$

$$= 2a^2(1 + \cos\varphi)$$

$$= 2a^2 \times 2\cos^2\varphi/2 = 4a^2\cos^2\frac{\varphi}{2}$$

I) Constructive interference

$$\text{Path Difference} = \frac{\lambda}{2\pi} \times 2n\pi = n\lambda$$

$$I_{max} = a_1^2 + a_2^2 + 2a_1a_2 = (a_1 + a_2)^2$$

$$= 4a^2$$

II) Destructive Interference

$$I_{min} = a_1^2 + a_2^2 - 2a_1a_2$$

$$= (a_1 - a_2)^2$$

$$= 0$$

Example 4.1 *Find the resultant of superposition of two waves $y_1 = 2.0 \sin\omega t$ and $y_2 = 5.0 \sin(\omega t + 30°)$. Symbols have their usual meanings.*

[GGSIPU, Dec. 2004, (4 marks)]

Solution. According to superposition principle,

Method I

According to superposition principle, we have $Y = y_1 + y_2$

$$Y = y_1 + y_2 = 2.0\sin(\omega t) + 5.0\sin(\omega t + 30°)$$

$$= 2.0\sin\omega t + 5.0(\sin\omega t\cos 30° + \cos\omega t \sin 30°)$$

$$= 2.0\sin\omega t + \frac{5.0\times\sqrt{3}}{2}\sin\omega t + \frac{5.0}{2}\cos\omega t$$

$$= (2.0 + 2.5\times 1.732)\sin\omega t + 2.5\cos\omega t$$

$$= 6.33\sin\omega t + 2.5\cos\omega t$$

$$= R\cos\theta\sin\omega t + R\sin\theta\cos\omega t$$

Here $R\cos\theta = 6.33$; $R\sin\theta = 2.5$

$$R^2(\sin^2\theta + \cos^2\theta) = 46.3189$$

$$R = 6.8$$

$$\tan\theta = \frac{R\sin\theta}{R\cos\theta} = 0.394$$

$$\theta = 21.55°$$

Then

$$Y = R\sin(\omega t + \theta)$$

$$= 6.8\sin(\omega t + 21.55°)$$

Method II

Given $a_1 = 2.0$, $a_2 = 5.0$, $\varphi = 30°$, the resultant amplitude

$$R = \sqrt{(a_1^2 + a_2^2 + 2a_1a_2\cos 30°)}$$

$$= \sqrt{4 + 25 + 2\times 2\times 5\times\sqrt{3}/2} = 6.8$$

and

$$\tan\theta = \frac{R\sin\theta}{R\cos\theta} = 0.394$$

then

$$\theta = 21.55°$$

Hence

$$Y = R\sin(\omega t + \theta)$$

$$= 6.8\sin(\omega t + 21.55°)$$

Problem 4.1 *Two waves of same frequency have amplitudes 1.00 and 2.00. They interference at a point, where the phase difference is 60°. What is the resultant amplitude ?* **[GGSIPU, Dec. 2009 (3 marks)]**

Solution. Given that $a_1 = 1.00$, $a_2 = 2.00$ and $\varphi = 60°$

We know that, the resultant amplitude

$$R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\varphi}$$

$$= \sqrt{1^2 + 2^2 + 2(1)(2)\cos 60°}$$

$$= \sqrt{1+4+2} = \sqrt{7} = 2.65 \text{ unit.}$$

Problem 4.2 *Superimpose the following waves*

$$y_1 = 20\sin\omega t\,; \quad y_2 = 20\sin(\omega t + 60°)$$

Show also the superimposition diagrammatically. **[GGSIPU, Dec. 2013 reappear (3 marks)]**

Solution. Given $a_1 = 20$, $a_2 = 20$ and $\varphi = 60°$

The resultant amplitude $\quad R = \sqrt{a_1^2 + a_2^2 + 2a_1a_2\cos\varphi}$

$$= \sqrt{(20)^2 + (20)^2 + 2\times20\times20\times\cos 60°}$$

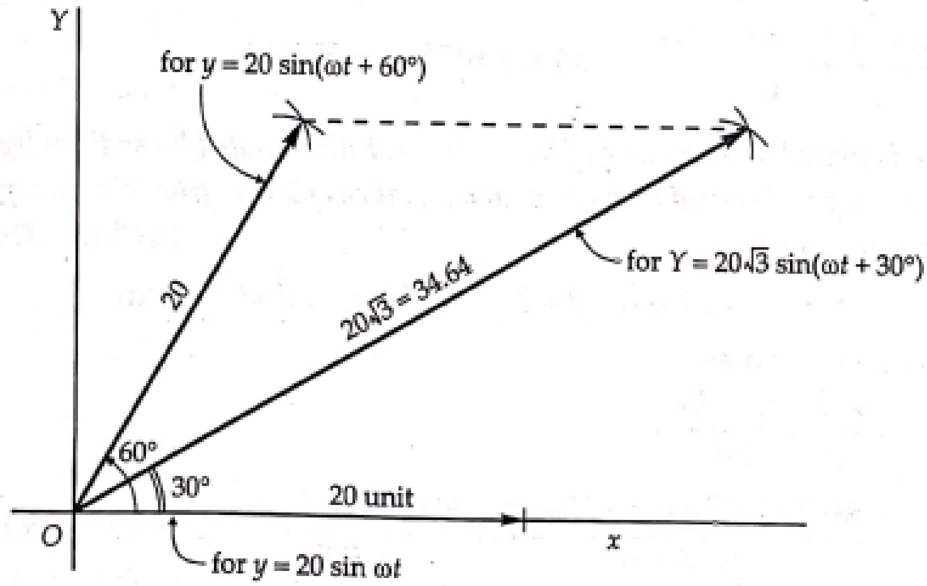
$$= \sqrt{400+400+400} = 20\sqrt{3} = 20\times1.732 = 34.64 = 35$$

Direction $\quad \tan\theta = \dfrac{a_2\sin\varphi}{a_1 + a_2\cos\varphi}$

$$= \frac{20\times\sin 60°}{20+20\cos 60°} = \frac{20\times\frac{\sqrt{3}}{2}}{20+20\times\frac{1}{2}} = \frac{1}{\sqrt{3}}$$

$$\theta = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = 30°$$

Resultant displacement $Y = 20\sqrt{3}\sin(\omega t + 30°)$ for $Y = 20\sqrt{3}\sin(\omega t + 30°)$



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Interference

Division of wave front

I) Young Double slit Exp.

II) Fresnel Biprism

Division of Amplitude

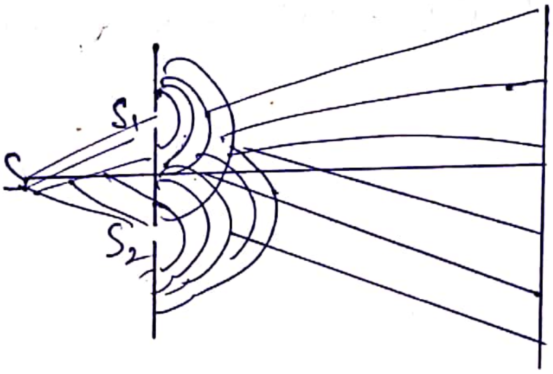
I) Thin Film

II) Newton's Ring

III) Michelson Interferometer

Division of wave front:-

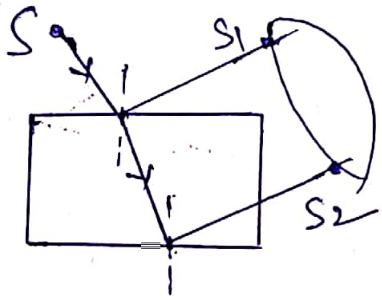
Wave front :→ Locus of all the particles of medium which are in same state of vibrations.



Reflection
Refraction
Diffraction.

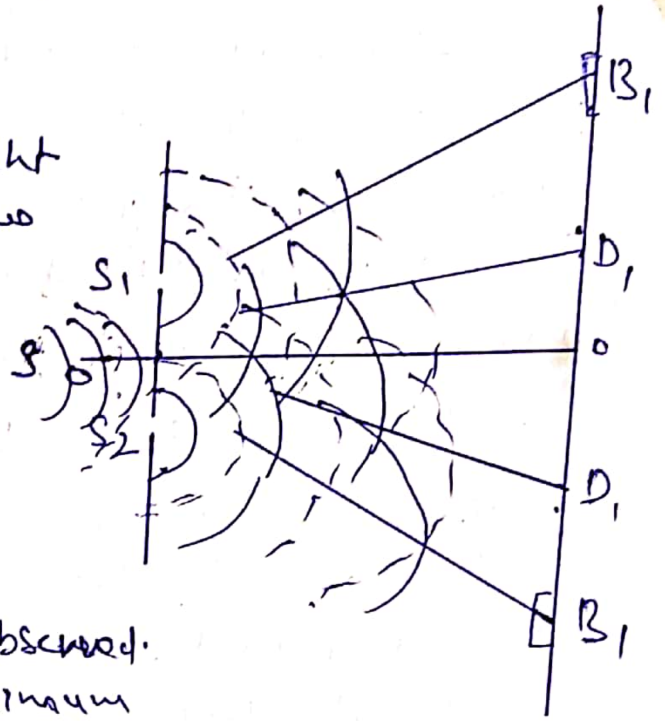
Division by Amplitude

Partial Reflection
Partial Refraction



Young's Double Slit Experiment:-

In 1801, Thomas Young demonstrated the interference light experimentally.



A source of monochromatic light S is used for illuminating two narrow slit S_1 & S_2. Two slits are very close to each other and at equal distance from source S.

The wave from from slit S_1 & S_2 spreadout in all directions and superimpose on the screen.

Alternate bright & Dark fringes observed.

At centre O intensity of light is maximum & known as central maxima.

As we move above & below the centre O alternate bright & dark fringes obtained.

From Youngs double slits Experiment following facts can be verified.

1) Interference pattern Disappear, if one of the two slit is closed. It shows that interference pattern is due to superposition of wave from two slits.

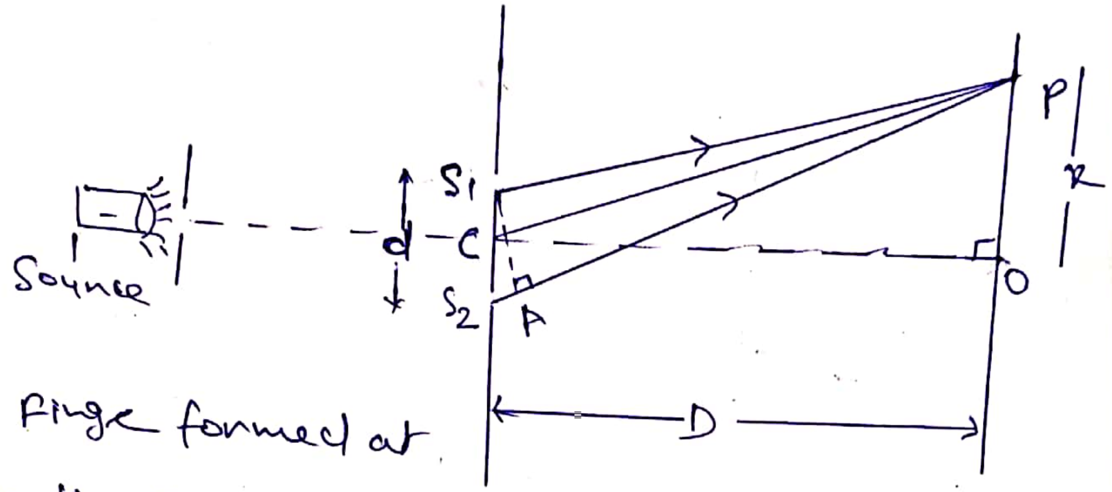
11) Instead of two slits illuminated with a single source, if two independent sources (S_1 & S_2) are used, the position of maximum & minimum intensity dont remain fixed. It shows that for producing interference coherent source (single source) should be used.

Condition for constructive interference:

$$\boxed{x = n\lambda} \quad (n = 0, 1, 2, 3 \ldots)$$

Condition for Destructive interference:-

$$\boxed{x = (2n+1)\frac{\lambda}{2}}$$



let a bright fringe formed at a Point P on the screen

$OP = x \qquad OC = D \qquad S_1S_2 = d$

Path Difference $= S_2P - S_1P = S_2A$

we have ΔS_1S_2A & ΔPCO are similar

$$\frac{S_2A}{S_1S_2} = \frac{OP}{CP}$$

& Substitute $CP = CO$ as d is very small as compare to OP

Then $$\frac{S_2A}{S_1S_2} = \frac{OP}{CO}$$

$$\Rightarrow \frac{S_2A}{d} = \frac{x}{D}$$

Path Diff. $$S_2A = \frac{xd}{D}$$

for constructive $\frac{xd}{D} = n\lambda \quad \Rightarrow x = \frac{nD\lambda}{d}$

for Destructive $\frac{xd}{D} = (2n-1)\frac{\lambda}{2}$

$$\Rightarrow x = \left(n - \frac{1}{2}\right)\frac{\lambda D}{d}$$

Fringe width:- let $x_n = \frac{nD\lambda}{d} \qquad x_{n+1} = (n+1)\frac{D\lambda}{d}$

$(x_{n+1} - x_n) = \frac{(n+1)D\lambda}{d} - \frac{nD\lambda}{d} = \frac{D\lambda}{d} \Rightarrow$ Fringe width $\beta = D\lambda/d$

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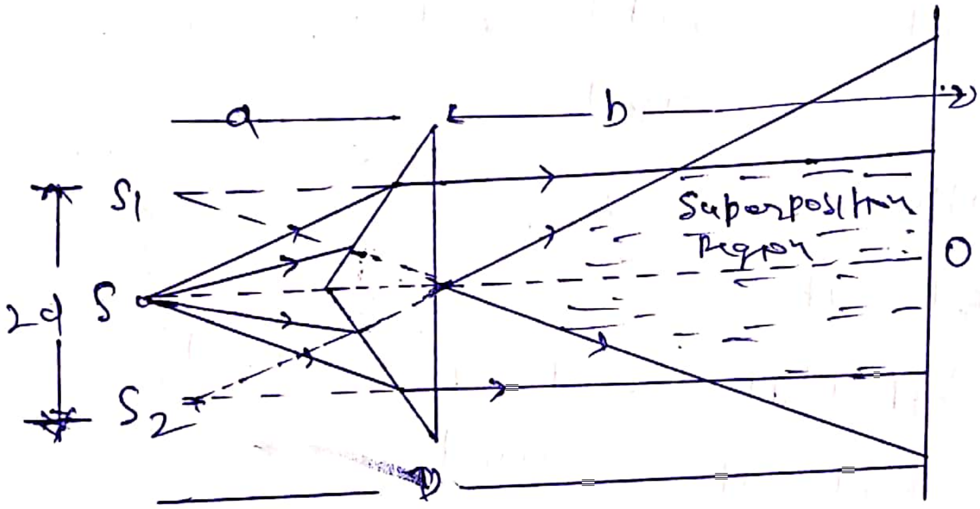
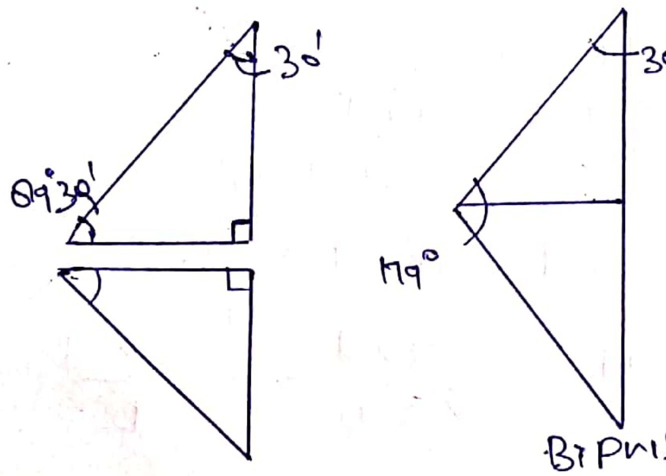
Frensel Bi Prism:-

It is an optical device which is used to produce two coherent sources of light by the phenomenon of refraction of light. We use division of wave front method to produce two coherent sources of light.





Biprism.

Problem 4.5 *A biprism is placed at a distance of 5 cm from slit illuminated by sodium light of wavelength 5890 Å. Find the width of fringes observed in eyepiece at a distance of 75 cm from biprism, given the distance between virtual sources is 0.005 cm.* **[GGSIPU, Oct. 2013 (2 marks)]**

Solution. Given $a = 5$ cm, $\lambda = 5890$ Å, $\beta = ?$, $b = 75$ cm, $2d = 0.005$ cm

The fringe width (β) is given as

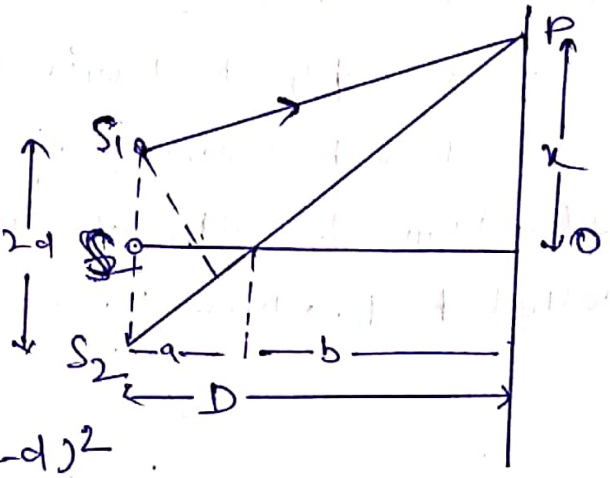
$$\beta = \frac{\lambda D}{2d} = \frac{\lambda(a+b)}{2d}$$

$$= \frac{5890 \times 10^{-8}\ \text{cm}\,(5+75)\ \text{cm}}{0.005\ \text{cm}} = \frac{5890 \times 10^{-8} \times 80}{0.005}\ \text{cm} = 589 \times 8 \times 10^{-3}\ \text{cm}$$

$$= 4.712\ \text{cm.}$$

Let O point be equidistant from S_1 & S_2

Then we have



$$(S_2P)^2 = D^2 + (x+d)^2$$

$$(S_1P)^2 = D^2 + (x-d)^2$$

$$(S_2P)^2 - (S_1P)^2 = (x+d)^2 - (x-d)^2$$

$$= 4xd$$

~~Now Path Difference =~~

~~$S_2P - S_1P =$~~

$$\Rightarrow (S_2P + S_1P)(S_2P - S_1P) = 4xd$$

$$(S_2P - S_1P) = \frac{4xd}{(S_2P + S_1P)}$$

Now Path Difference =

$$S_2P - S_1P = \frac{4xd}{S_2P + S_1P}$$

We Assume ($S_2P = S_1P = D$) then.

$$S_2P - S_1P = \frac{4xd}{2D} = \frac{2xd}{D}$$

For constructive

$$\frac{2xd}{D} = n\lambda \Rightarrow x = \frac{D}{2d} n\lambda$$

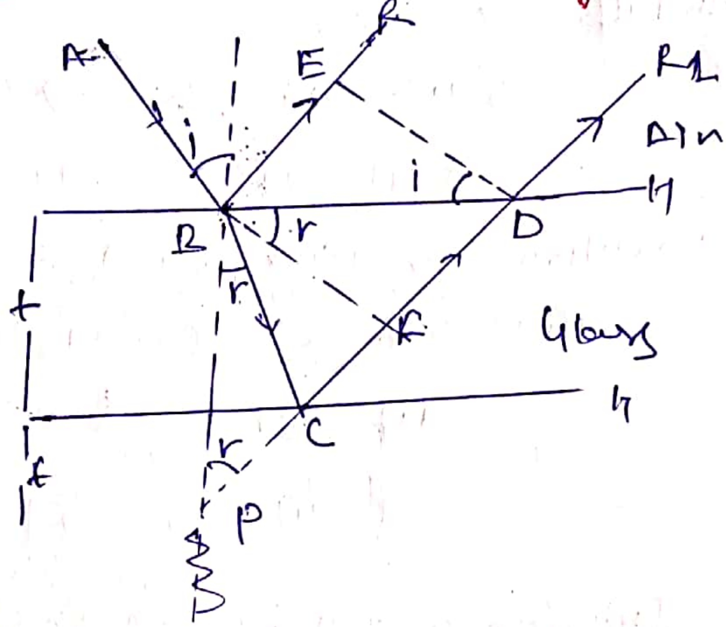
Fringe width. $\beta = x_{n+1} - x_n$

$$= \frac{D}{2d}(n+1)\lambda - \frac{D}{2d} n\lambda$$

$$= \frac{D\lambda}{2d} \Rightarrow \cancel{\lambda = \frac{xd}{D}\beta}$$

$$\Rightarrow \lambda = \frac{2d}{D}\beta = \frac{2d}{(a+b)}\beta$$

Interference due to thin film :- 1) Reflected light



Consider a transparent thin film of thickness t

A ray AB be incident on the upper surface of thin film

AB is partly reflected along BR & partly refracted along BC.

At C again partly reflected along CD.

This process continued throughout the thin film.

Path difference between reflected ray BR & DR₁ can be calculated. Draw ED normal to BR and BF to CD

Angle of incidence is i and angle of refraction is r.

Optical path difference between two reflected rays (BR & DR₁) is given by:-

$$\Delta = \text{Path } (BC+CD) \text{ in film} - \text{path } BE \text{ in Air.}$$

$$= \mu (BC+CD) - BE \quad \text{—(1)}$$

$$\mu = \frac{\sin i}{\sin r} = \frac{BE/BD}{FD/BD} = \frac{BE}{FD}$$

$$BE = \mu (FD) \quad \text{—(2)}$$

Put (2) in (1)

$$\Delta = \mu (BC+CD) - \mu (FD)$$

$$= \mu (BC + CF + \cancel{FD} - \cancel{FD}) = \mu (BC+CF)$$

$$\Rightarrow \Delta = \mu (PC + CF) = \mu PF \quad \text{—(3)} \qquad \because (PC = BC)$$

Now in $\triangle BPF$ $\cos r = \frac{PF}{BP} \Rightarrow PF = BP \cos r$.
$= 2t \cos r$ —(4)

Equating (3), (4) we get $\Delta = \mu 2t \cos r = 2\mu t \cos r$

Condition for Bright Band

Path Diff. $\Delta = n\lambda$

$$2\mu t \cos r \pm \frac{\lambda}{2} = n\lambda$$

$$2\mu t \cos r = (2n\pm1)\frac{\lambda}{2}$$

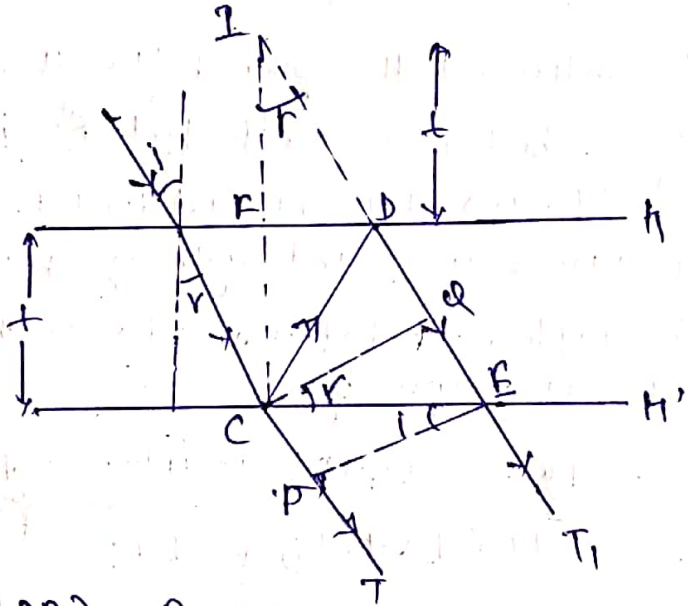
Condition for Dark Band.

Path Diff $\Delta = (2n\pm1)\lambda/2$

$$2\mu t \cos r \pm \frac{\lambda}{2} = (2n\pm1)\frac{\lambda}{2}$$

$$\boxed{2\mu t \cos r = n\lambda}$$

<u>Interference due to Transmitted light:-</u>



Effective Path Difference

$$\Delta = \mu(CD+DE) - CP \quad (1)$$

$$\mu = \frac{\sin i}{\sin r} = \frac{CP/CE}{QE/CE} = \frac{CP}{QE} \Rightarrow CP = \mu(QE) \quad (2)$$

from (1), (2)

$$\Delta = \mu(CD+DQ+QE) - QE(\mu)$$

$$= \mu(CD+DQ) \qquad \because (CD = ID)$$

$$= \mu(IQ)$$

$$\Delta = \mu\, 2t\cos r \Rightarrow \boxed{\Delta = 2\mu t \cos r}$$

Bright - $\Delta = 2\mu t \cos r = n\lambda$

Dark - $\Delta = 2\mu t \cos r = (2n\pm1)\frac{\lambda}{2}$

Example 4.11 *A soap film, suspended in air has thickness 5×10^{-5} cm and viewed at an angle 35° to the normal. Find the wavelength of light in visible spectrum, which will be absent for a reflected light. The μ for the soap film as 1.33 and the visible spectrum is 4000 to 7800 Å* **[GGSIPU, Dec. 2009 (4 marks)]**

Solution. In colour thin film :

Given that : $t = 500 \text{ nm} = 5.0 \times 10^{-7}$ m, $i = 35°$, $\mu = 1.33$

We know that :

$$2\mu t \cos r = n\lambda \quad \text{...}(i)$$

and
$$\mu = \frac{\sin i}{\sin r}$$

$$\Rightarrow \quad \sin r = \frac{\sin 35°}{1.33}$$

then
$$\cos r = \sqrt{(1 - \sin^2 r)} = \left[1 - \left(\frac{\sin 35°}{1.33}\right)^2\right]^{1/2} = \sqrt{(1 - 0.186)} = 0.902$$

For *first order i.e., n* = 1.

$$\lambda_1 = 2\mu t \cos r = 2 \times 1.33 \times 5.0 \times 10^{-7} \times 0.902$$
$$= 1.199 \times 10^{-6} \text{ m} = 12000 \text{ Å (approx.)}$$

For *second order i.e., n* = 2

$$\lambda_2 = \frac{2\mu t \cos r}{2} = \mu t \cos r = 1.33 \times 5.0 \times 10^{-7} \times 0.902 = 6000 \text{ Å (approx.)}$$

For *third order i.e., n* = 3

$$\lambda_3 = \frac{2\mu t \cos r}{3} = \frac{2 \times 1.33 \times 5.0 \times 10^{-7} \times 0.902}{3} = 4000 \text{ Å (approx.)}$$

For *fourth order i.e., n* = 4

$$\lambda_4 = \frac{2\mu t \cos r}{4} = \frac{2 \times 1.33 \times 5.0 \times 10^{-7} \times 0.902}{4} = 3000 \text{ Å (approx.)}$$

Hence λ_2 and λ_3 wavelengths of light in visible spectrum will be absent.

Interference due to wedge Shape thin Film:-

Thin Film:- Layer of Material deposited on a surface which decide its Properties known as thin Film.

Wedge Shape thin film:- A film of Variable thickness is known as wedge shape thin film.

OR

A thin Film of varying thickness having zero thickness at one Point and Progressively increasing to a Particular thickness at other end is known as wedge.

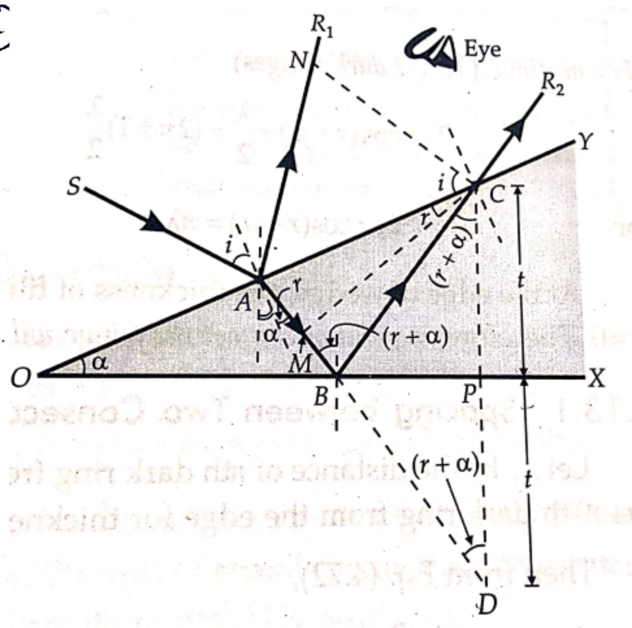
Let us consider OX & OY are two planes inclied at an angle α. YOX is the region inside thin film. μ is the refractive index inside YOX.

Then the Path Difference (Δ) between ray AR_1 & CR_2 is

$$\Delta = (AB+BC)_{Med} - (AN)_{Air}$$

$$= \mu(AM+MB+BC) - (AN)$$

$$= \mu(AM+MB+BC) - (AN) \quad (1)$$



Interference produced by wedge shaped film.

from Snell's law

$$\mu = \frac{\sin i}{\sin r}$$

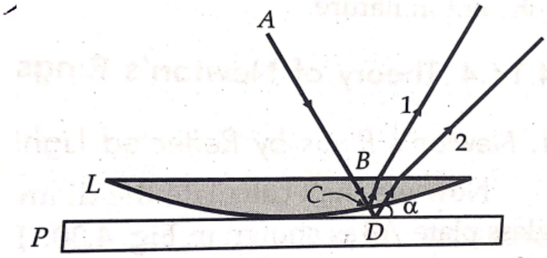
from ΔANC & AMC,

$$\mu = \frac{AN/AC}{AM/AC} \Rightarrow AN = \mu AM \quad (2)$$

If refractive index of Medium (Film) is greater than the refractive index of incident ray (AR_1) then AR_1 suffer Path diff. of $\lambda/2$ then (1) becomes-

$$\Delta = \mu(AM+MB+BC) - \left(AN \pm \frac{\lambda}{2}\right)$$

Substituting ② in ①



$$\Delta = \mu(AM+MB+BC-AM) \pm \frac{\lambda}{2}$$

$$\Delta = \mu(MB+BC) \pm \frac{\lambda}{2} \quad -③$$

Also we have $BC = BD$ & $CP = PD = t$

then ③ becomes

$$\Delta = \mu(MB+BD) \pm \frac{\lambda}{2} = \mu MD \pm \frac{\lambda}{2}$$

From ΔCMD,

$$MD = 2t\cos(r+\alpha)$$

$$\Rightarrow \boxed{\Delta = 2\mu t\cos(r+\alpha) \pm \frac{\lambda}{2}}$$

I) For maxima

$$2\mu t\cos(r+\alpha) \pm \frac{\lambda}{2} = n\lambda$$

$$\Rightarrow \boxed{2\mu t\cos(r+\alpha) = (2n \pm 1)\frac{\lambda}{2}}$$

II) For minima

$$2\mu t\cos(r+\alpha) \pm \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

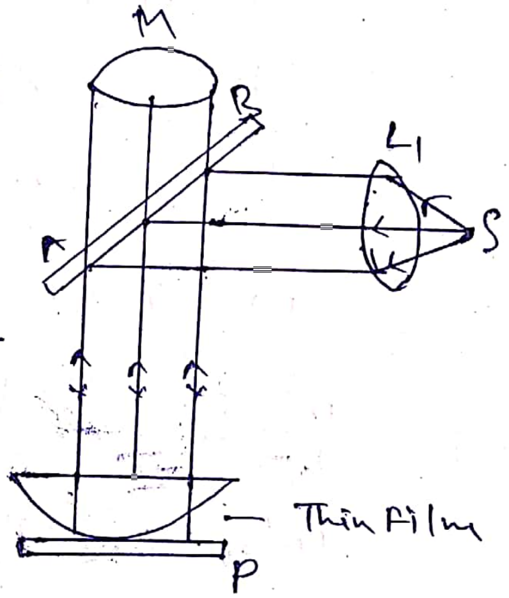
$$\Rightarrow \boxed{2\mu t\cos(r+\alpha) = n\lambda}$$

Newton Ring:- (Interference by wedge shape film)

A pattern of interference produced by the contact of convex surface of lens with a plane glass plate, appearing as a series of concentric, alternate bright & dark ring.

or

"A series of alternate bright & dark ring that appear when a convex lens comes into the contact of glass plate."



→ A plano convex lens is placed on a glass P in such a way that curved surface of glass touches the glass plate.

→ Thin film is formed between lower surface of lens. (wedge shape film)

→ If a monochromatic light falls on the film a set of alternate bright & dark fringes will be seen in the film.

→ Interference occur between the ray reflected from the upper surface and lower surface of film.

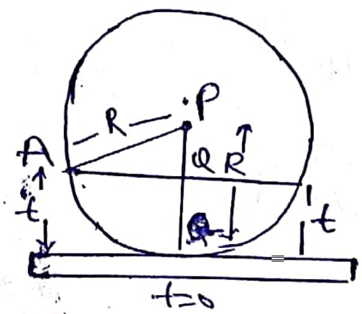
Now In ΔAPQ we have

$(AP)^2 = (PQ)^2 + (AQ)^2$

$R^2 = (R-t)^2 + (AQ)^2$

$\cancel{R^2} = \cancel{R^2} + t^2 - 2Rt + (AQ)^2$

$(AQ)^2 = 2Rt - t^2$



Let t be small then t^2 will becomes very small

$\Rightarrow t^2 = 0$ let $AQ = r_n$

$(AQ)^2 = 2Rt$

$\Rightarrow r_n^2 = 2Rt \Rightarrow \boxed{2t = \frac{r_n^2}{R}}$ —(1)

Now we have Path Difference in case of wedge shape thin film =

$$\Delta = 2\mu t \cos(r+\alpha) \pm \frac{\lambda}{2}$$

Effective Path Difference will be

$(\mu = 1, r = 0, \alpha = 0$, for Air$)$

$$\Rightarrow \boxed{\Delta = 2t \pm \frac{\lambda}{2}}$$

For n^{th} Bright fringes we have

$$2t - \frac{\lambda}{2} = n\lambda$$

$$2t = \frac{(2n+1)\lambda}{2} \quad \text{—(2)}$$

Comparing (1) & (2)

$$\frac{r_n^2}{R} = \frac{(2n+1)\lambda}{2}$$

$$r_n^2 = \frac{(2n+1)\lambda R}{2}$$

then Diameter of Ring

$$\left(\frac{D_n}{2}\right)^2 = \frac{(2n+1)\lambda R}{2}$$

$$D_n^2 = \frac{4(2n+1)\lambda R}{2} = 2(2n+1)\lambda R$$

$$(D_{n+p})^2 = 2(2(n+p)+1)\lambda R$$

$$(D_{n+p})^2 - D_n^2 = 2(2(n+p)+1)\lambda R - 2(2n+1)\lambda R$$

$$= 2\lambda R\,(2n+2p+1-2n-1)$$

$$= 4pR\lambda$$

$$\Rightarrow \boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$

Similarly Refractive Index can be calculated

$$\mu = \frac{(D_{n+p}^2 - D_n^2)_{Air}}{(D_{n+p}^2 - D_n^2)_{Liquid}}$$

Now for nth Dark Ring we have

$$2t - \frac{\lambda}{2} = (2n \pm 1)\frac{\lambda}{2}$$

$$\Rightarrow 2t = n\lambda$$

From (1) $\quad \frac{r_n^2}{R} = n\lambda$

$$r_n^2 = n\lambda R$$

$r_n = \frac{D_n}{2} \quad \Rightarrow \quad \boxed{D_n = 2\sqrt{n\lambda R}}$

Hence Diameter $D = 2\sqrt{n\lambda R}$

Raddi $= \sqrt{n\lambda R}$

Raddi of Curvature of lens

$$\boxed{R = \frac{D_n^2}{4n\lambda}}$$

(2) Newton Ring by Transmitted light

Path Difference $\Delta = 2\mu t \cos r$

$(\mu = 1, r = 0, \alpha = 0$ for Air$)$

I) For maxima (Bright) $\quad 2t = n\lambda$

$$\Rightarrow \boxed{D = 2\sqrt{n\lambda R}}$$

II) For Minima $\quad 2t = (2n \pm 1)\lambda/2$

$$\Rightarrow \boxed{D = \sqrt{2\lambda R}\sqrt{(2n \pm 1)}}$$

Numerical:

Q In Newton Ring Experiment the Diameter of 15th Ring was found to be 0.590 cm & 5th ring was 0.336 cm. If the Radius of curvature of plano convex lens is 100 cm Calculate the wavelength of light used.

Sol.

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR}$$

$\Rightarrow D_{n+p} = D_{15} = 0.590$ CM $\quad D_n = D_5 = 0.336$ CM.

$P = 10 \quad R = 100$ CM

$$\lambda = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 100}$$

$$= 5882 \times 10^{-8} \text{ CM}$$

$$= 5882 \text{ A}^\circ.$$

Q In Newton Ring Experiment Diameter of a third dark Ring is 3.2 mm. Find the Radius of Curvature of the lens if λ of light $= 5890 \times 10^{-8}$ CM.

Sol. Diameter of Dark Ring is given by -

$$D_n^2 = 4n\lambda R$$

$D_n = D_3 = 3.2$ mm $= 0.32$ CM, $n = 3$

$\lambda = 5890 \times 10^{-8}$ cm.

$$R = \frac{D_n^2}{4n\lambda} = \frac{(0.32)^2}{4 \times 3 \times 5890 \times 10^{-8}} = 145 \text{ cm}$$

Q In Newton Ring Experiment Diameter of 5th Ring was 0.336 CM and Diameter of 15th Ring was 0.590 cm. Find the Radius of Curvature of plano convex lens if $\lambda = 5890$ A°

Sol^n $D_{n+p} = D_{15} = 0.590$ CM, $D_n = D_5 = 0.0336$ CM

$P = 10$, $\lambda = 5890 \times 10^{-8}$ CM

$$\lambda = \frac{D_{n+p}^2 - D_n^2}{4PR} \Rightarrow R = \frac{D_{n+p}^2 - D_n^2}{4P\lambda}$$

$$R = \frac{(0.590)^2 - (0.336)^2}{4 \times 10 \times 5890 \times 10^{-8}} \quad 99.83 \text{ cm}$$

Example 4.14 *In a Newton's ring experiment the diameters of 4th and 12th dark rings are 0.4 cm and 0.8 cm respectively. Deduce the diameter of 20th dark ring.* **[GGSIPU, Dec. 2011 ; Dec. 2012 (2.5 marks)]**

Solution. In Newton's ring experiment,

Given that : $n=4$; $(m+n)=12$, $m=8$

$$D_n = 0.4 \text{ cm and } D_{m+n} = 0.8 \text{ cm}$$

The wavelength of sodium light using Newton's ring is

$$\lambda = \frac{D_{m+n}^2 - D_n^2}{4mR}$$

or
$$4\lambda R = \frac{D_{m+n}^2 - D_n^2}{m}$$

$\Rightarrow$
$$4\lambda R = \frac{(0.8)^2 - (0.4)^2}{m} \qquad ...(i)$$

We know that the diameter of nth dark ring in presence of air is

$$D_n^2 = 4n\lambda R$$

$\Rightarrow$
$$D_{20}^2 = 20 \times (4\lambda R) \qquad ...(ii)$$

Putting the value of $4\lambda R$ from Eq. (i) in Eq. (ii)

$$D_{20}^2 = \frac{20 \times [(0.8)^2 - (0.4)^2]}{8} = \frac{20}{8} \times 1.2 \times 0.4 \quad \Rightarrow \quad D_{20} = 1.2 \text{ cm}$$

<u>***Problem 4.11***</u> *A Newton ring arrangement is used with a light sources of wavelength $\lambda_1 = 6000$ Å and $\lambda_2 = 5000$ Å and it is found that the nth dark ring due to λ_1 coincide with $(n+1)$th dark ring due to λ_2. If the radius of curvature of curved surface of the lens is 90 cm, then find the diameter for the nth, dark ring for λ_1.*

[GGSIPU, Sept. 2009 (3 marks)]

Solution. Given $\lambda_1 = 6000$ Å for nth ring

$$\lambda_2 = 5000 \text{ Å for } (n+1)\text{th ring, } R = 90 \text{ cm} = 0.9 \text{ m}$$

and
$$(D_n)_{\lambda_1} = (D_{n+1})_{\lambda_2} \qquad [\because D_n = \sqrt{4n\lambda R}]$$

or
$$\sqrt{4n\lambda_1 R} = \sqrt{4(n+1)\lambda_2 R}$$

or
$$n\lambda_1 = (n+1)\lambda_2$$

$\Rightarrow$
$$6000 \times n = (n+1) \times 5000$$

$\Rightarrow$
$$n = 5$$

$$D_n = \sqrt{4n\lambda_1 R}$$

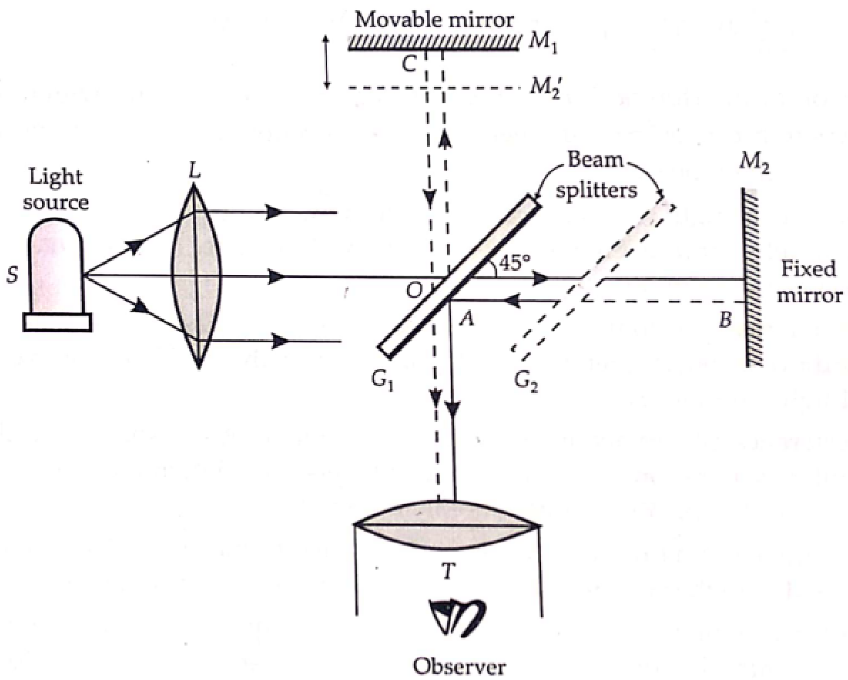
$$D_5 = \sqrt{4 \times 5 \times 6000 \times 10^{-10} \times 0.9} = 3.286 \times 10^{-3} \text{m}$$

Michelson Interferometer:-

An Interferometer is an instrument in which the Phenomenon of interference is used to make precise measurement of wavelength or Distance.

Michelson designed an interferometer which utilize the thin film interference.

Principle:- In Michelson Interferometer a beam of light form an extended source divided into two parts of Equal intensities by partial reflection & refraction. Beam travel in mutually perpendicular direction after Reflection from the Mirror. Beam overlap each other and produce Interference fringes.



* There are two parallel glass plate G_1 & G_2 of same thickness. Glass plate G_1 is semi silvered on the backside and function as a beam splitter.
* Function of compensating plate G_2.

To equalise the path of AC & AB G_2 is used also called compensating plate

Path Difference

$$\Delta = 2t\cos\theta$$

I) Condition for Brightness

$$2t\cos\theta = n\lambda$$

II) Condition for Dark

$$2t\cos\theta = (2n \pm \frac{\lambda}{2})$$

Note: θ – Angle θ, The observer look into the system at an angle θ.

t – Thickness of film.

Applications: Measurement of wave length of light can be determined by Michelson interferometer

$$\cancel{\lambda = \frac{2t}{N}} \quad \boxed{\lambda = \frac{2t}{N}}$$

N – Fringes

II) To determine the difference in two waves.

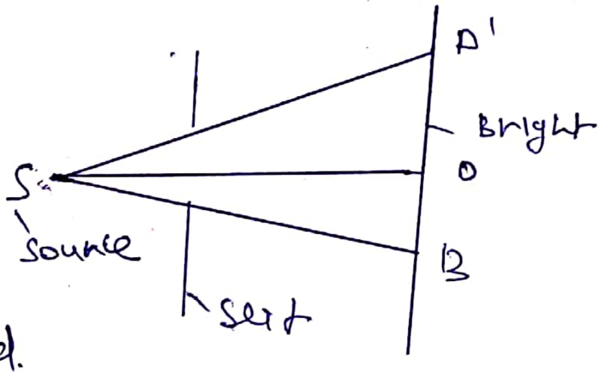
$$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda^2}{2t}$$

III) Thickness of thin Transparent sheet

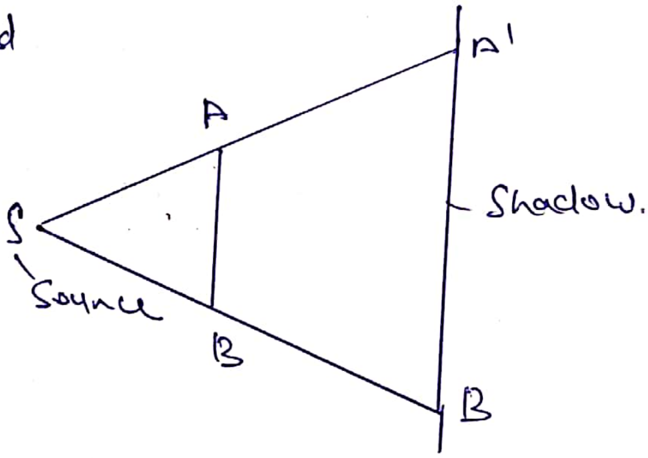
$$t = \frac{m\lambda}{2(\mu - 1)}$$

Diffraction :- The phenomenon of bending of light round the sharp corner and spreading into the region of geometrical shadow. is called Diffraction of light

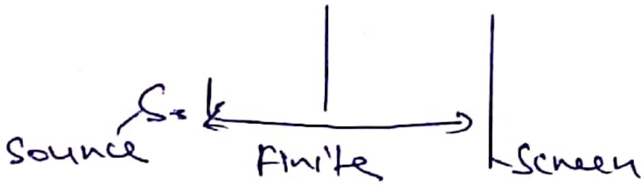
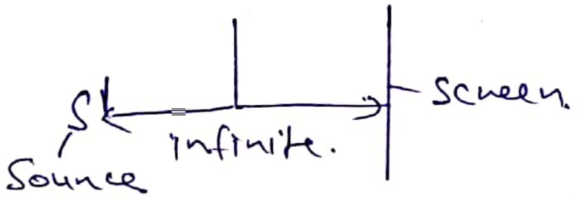
1) When a narrow slit is placed in the path of light only the region A'B' on the screen should get illuminated.



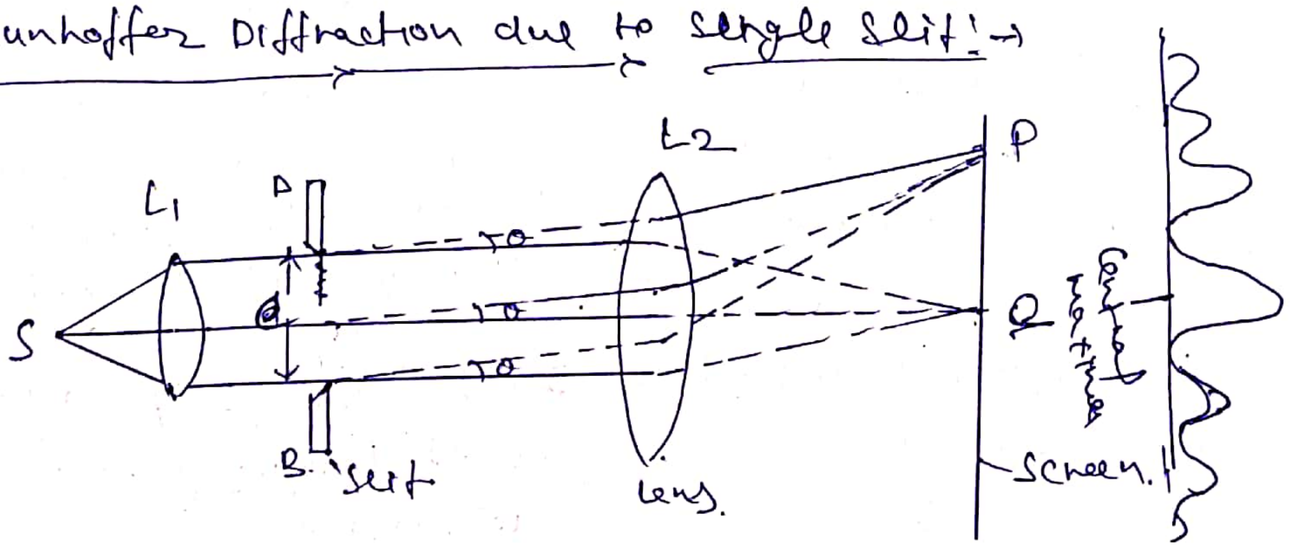
II) When an obstacle AB is placed in the path of light, then its distinct geometrical shadow should be obtained on the screen.



Interference	Diffraction
1) It occurs due to superposition of secondary wavelets from ~~two~~ coherent sources of light.	1) It occurs due to superposition of secondary wavelets from exposed part of single source.
2) All bright fringes have same intensity	3) Intensity of successive bright fringes goes on decreasing.
3) Fringes due to mono-chromatic light has same width	3) Diffraction fringes are never of equal width
4) Intensity of all dark fringes are zero	4) Intensity of dark fringes are not zero.

Fresnel Diffraction.	Fraunhoffers Diffraction.
I) In Fresnel Diffraction Source, Screen & diffraction device are at finite Distance	I) In Fraunhoffers Diffraction Source, Screen and Diffraction device are at Infinite Distance
	
II) No lens & No Mirror Used	II) At infinite distance intensity of light becomes low. Hence lens (convex) is used.
III) center may be dark or bright	III) Center always bright
IV) wavefront are spherical or cylindrical	IV) Plane wavefront are used.
V) Diffraction device are zone plate, ~~etc~~ circular Ring etc.	V) Diffraction device are Double slit, n slit, grating etc.

Fraunhoffer Diffraction due to single slit :→



* A Monochromatic Source of light S, emitting light wave of wavelength λ is placed at the Principle focus of Convex lens L_1
* Diffraction pattern obtained on screen lying at a distance D from the slit. ~~after~~ ~~Convex lens~~

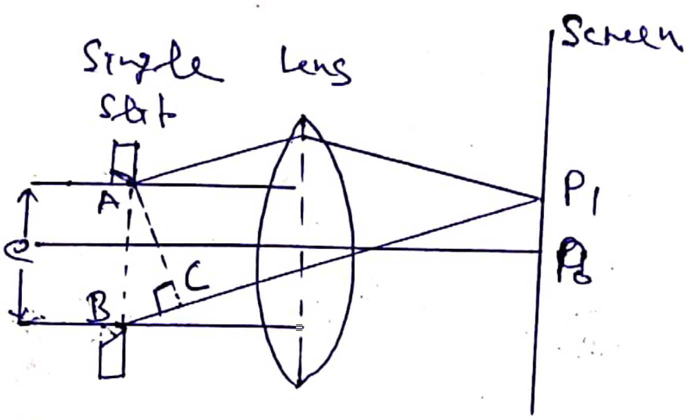
Path Difference = BC

$= e\sin\theta$

Phase Diff $= \frac{2\pi}{\lambda}$ Path diff

$= \frac{2\pi}{\lambda}(e\sin\theta)$

$$\boxed{\delta = \frac{1}{n}\frac{2\pi}{\lambda}(e\sin\theta)} \quad — (1)$$



From the theory of H- harmonic vibrations. we have, Resultant Amplitude R.

$$R = \frac{a\sin\frac{n\delta}{2}}{\sin\delta/2}$$

$$= \frac{a\sin\frac{\cancel{n}}{\cancel{2}}\left(\frac{1}{\cancel{n}}\frac{\cancel{2}\pi}{\lambda}e\sin\theta\right)}{\sin\frac{1}{\cancel{2}}\left(\frac{1}{n}\frac{\cancel{2}\pi}{\lambda}e\sin\theta\right)}$$

$$= \frac{a\sin\frac{\pi}{\lambda}e\sin\theta}{\sin\frac{\pi}{n\lambda}e\sin\theta}$$

Let us consider $\frac{\pi}{\lambda}e\sin\theta = \alpha$

$$\Rightarrow \boxed{R = \frac{a\sin\alpha}{\sin\left(\frac{\alpha}{n}\right)}}$$

if $\frac{\alpha}{n}$ is very small angle

$$\Rightarrow R = \frac{a\sin\alpha}{\frac{\alpha}{n}} = \frac{na\sin\alpha}{\alpha}$$

$$\boxed{R = \frac{A\sin\alpha}{\alpha}} \quad — (2)$$

Scanned with CamScanner

Intensity is Related to Amplitude

Intensity = $(\text{Amplitude})^2$

$$\boxed{I = \frac{A^2 \sin^2 \alpha}{\alpha^2}}$$

I) Condition for Minima $I = 0 \Rightarrow \frac{A^2 \sin^2 \alpha}{\alpha^2} = 0$

$\Rightarrow \sin \alpha = 0$

$\alpha = \pm n\pi$

Hence $\pm 1\pi, \pm 2\pi, \pm 3\pi$ ----

II) Condition for Maxima

I should be Maximum.

ie $I = \infty$

$$\frac{A^2 \sin^2 \alpha}{\alpha^2} = \infty$$

Means $\alpha = 0$ will give Maximum Intensity.

Scanned with CamScanner

Fraunhofer's Diffraction due to two slits :-

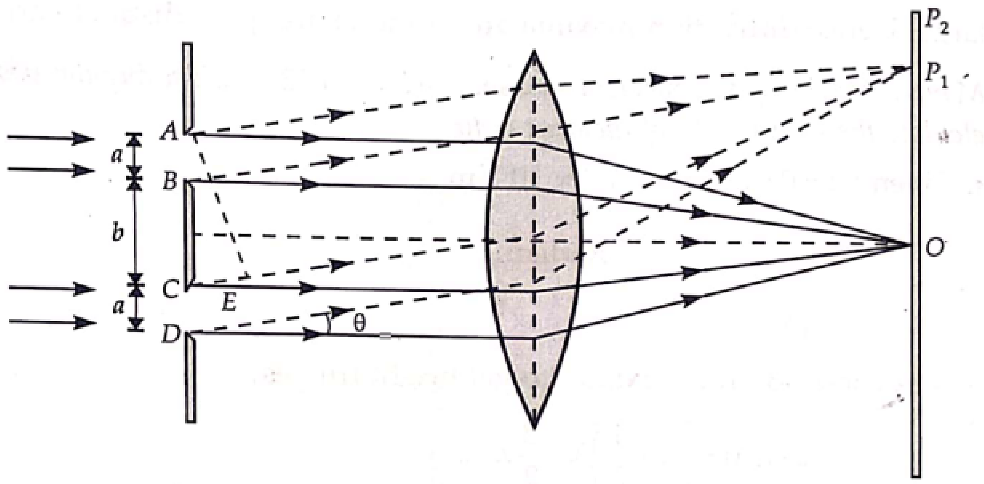
Let us consider a parallel beam of monochromatic light of wavelength λ incident on double slit and each slit diffract at an angle θ.

Then Path difference (~~at B & C~~) (at B & C)

$$CE = (a+b)\sin\theta$$

We have Phase Difference $= \frac{2\pi}{\lambda} \times$ Path difference

$$2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta \quad \text{—(1)}$$



Fraunhofer's diffraction at double slit.

Here each slit can be treated as an independent small source of light. Resultant pattern will be same as due to two sources.

If 2α is the phase difference between the extreme rays from first slit, then

$$2\alpha = \frac{2\pi}{\lambda} a\sin\theta$$

$$\alpha = \frac{\pi}{\lambda} a\sin\theta \quad \text{—(2)}$$

The resultant displacement y_1 due to the rays from the first slit is given by -

$$y_1 = A\sin\omega t \quad \text{-(3)}$$

Where $A = A_0 \frac{\sin\alpha}{\alpha}$ (Amplitude)

A_0 is the Amplitude of the direct ray

A is the Amplitude of diffracted ray at angle θ from first slit.

Then Resultant displacement y_2 due to two rays from the second slit is given by -

$$y_2 = A\sin(\omega t + 2\beta) \quad \text{-(4)}$$

Then Resultant Displacement Y due to rays from two slit diffracted at angle θ is given by -

$$Y = y_1 + y_2$$

$$= A\sin\omega t + A\sin(\omega t + 2\beta)$$

$$= A(\sin\omega t + \sin(\omega t + 2\beta))$$

$$\cancel{= 2A\cos\beta\sin(\omega t+\beta)}) = 2A\cos\beta\sin(\omega t+\beta) \quad \text{-(5)}$$

Substituting (5) in (4)

Then Resultant Amplitude,

$$R = 2A\cos\beta$$

Substituting $A = A_0 \frac{\sin\alpha}{\alpha}$ from (3)

$$\Rightarrow R = 2A_0 \frac{\sin\alpha}{\alpha}\cos\beta \qquad \text{But } I \propto A^2$$

$$\Rightarrow I \propto 4A_0^2 \frac{\sin^2\alpha}{\alpha^2}\cos^2\beta$$

Suppose constant of proportionality is 1 then

$$I = 4A_0^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \cos^2\beta$$

Now 1) Condition for maxima.

I will be maximum when $\sin\alpha = 0$

$$\sin\alpha = 0$$

$$\alpha = \pm m\pi$$

As we have $\alpha = \dfrac{\pi a \sin\theta}{\lambda}$

$$\Rightarrow \frac{\pi a \sin\theta}{\lambda} = \pm m\pi$$

$$\pi a \sin\theta = \pm m\pi\lambda$$

$$\boxed{a\sin\theta = \pm m\lambda}$$

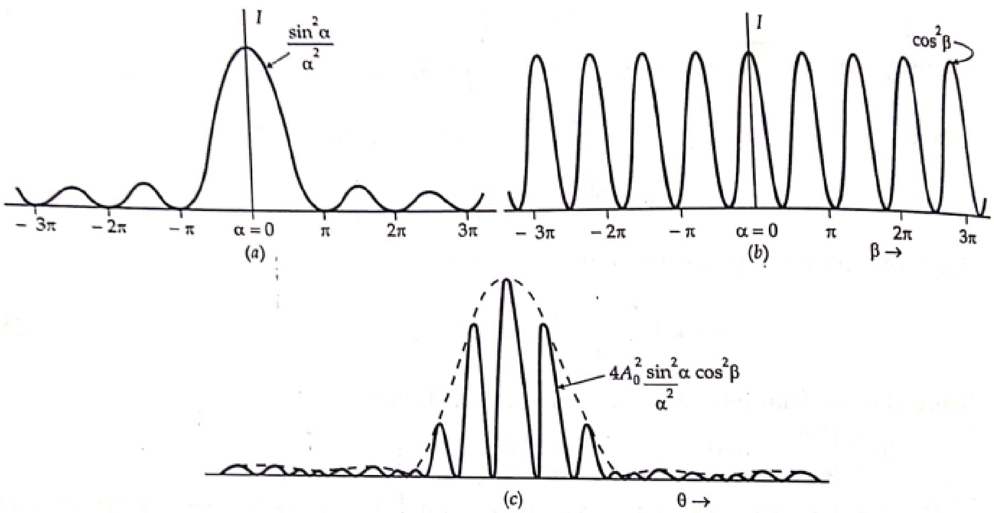
Similarly for maxima

$$\cos^2\beta = 1$$

$$\beta = \pm n\pi$$

$$\frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$$

$$\boxed{(a+b)\sin\theta = \pm n\lambda}$$



Resultant intensity distribution pattern of double slit.

Fraunhoffer Diffraction due to N Slits

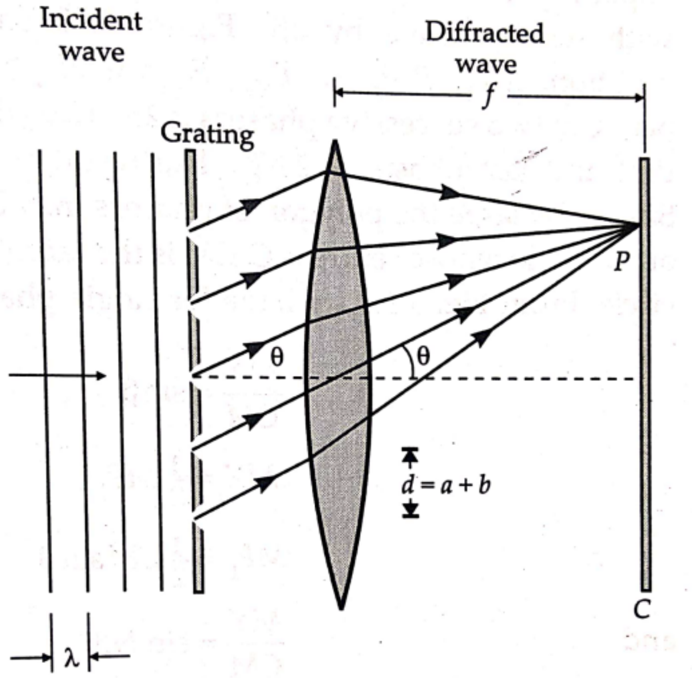
"An arrangement consist of large Number of parallel equidistant narrow rectangular Slits of Same width is known as diffraction grating"

we have considered-

I) Each Slit is of width a and has Same length

II) All Slits are parallel to each other.

III) Space between two Slit is Same.



Fraunhofer diffraction of a plane wave incident normally on a multiple slit aperture.

Consider point P on the screen where diffracted waves at angle θ will Superimpose.

From theory of diffraction all the points of slit can be summed up into single wavelet of Amplitude A.

$$A = A_0 \frac{\sin \alpha}{\alpha} \quad -(1)$$

If 2α is the phase difference between two extreme rays then

$$2\alpha = \frac{2\pi}{\lambda} a \sin\theta$$

$$\alpha = \frac{\pi}{a} a \sin\theta \quad -(2)$$

The path difference from two nearby slit is given by-

$$\Delta = (a+b)\sin\theta \quad -(3)$$

The we have phase Difference $= \frac{2\pi}{\lambda} \times$ Path Difference

$$\Rightarrow \quad 2\beta = \frac{2\pi}{\lambda}(a+b)\sin\theta \quad -(4)$$

Now to find resultant Intensity (I) we have to superimpos N waves each of Amplitude A with phase difference of 2β with nearby wave.

Then we have

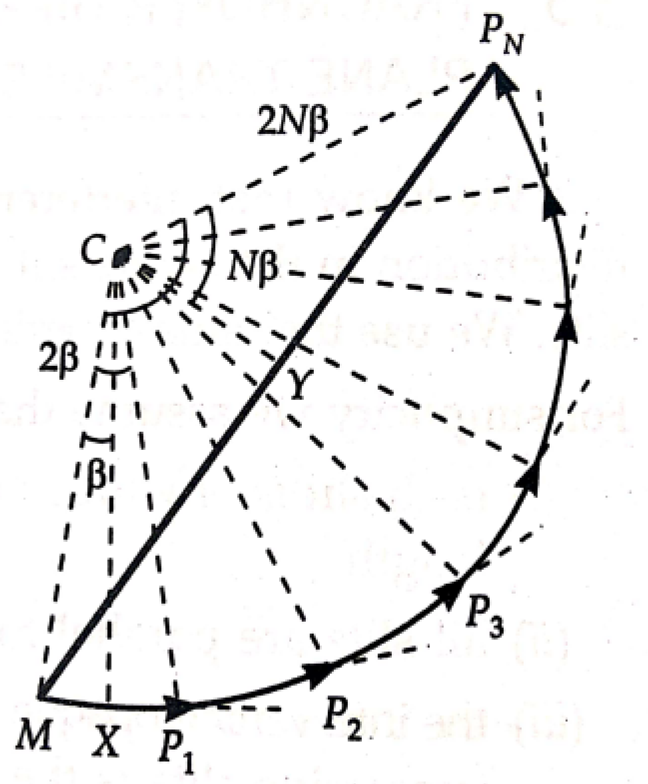
$$\frac{MX}{CM} = \sin\beta$$

$$\Rightarrow \quad MX = CM\sin\beta$$

But $MX = \frac{1}{2}MP_1$

$$\Rightarrow \frac{1}{2}MP_1 = CM\sin\beta$$

or $MP_1 = 2CM\sin\beta \quad -(5)$



Phasor diagram for *N* slits.

Also $\frac{MY}{CM} = \sin N\beta$

$$\Rightarrow \quad MY = CM\sin N\beta$$

But $MY = \frac{1}{2}MP_N$

$$\frac{1}{2}MP_N = CM\sin N\beta$$

$$MP_N = 2CM\sin\beta \quad -(6)$$

Dividing (6) by (5) we have

$$\frac{MP_N}{MP_1} = \frac{\sin N\beta}{\sin\beta}$$

MP_N – Resultant Disturbance of N Slit at θ

MP_1 – Single slit Disturbance

Hence

$$R_o = \frac{A \sin N\beta}{\sin\beta} \quad \text{where } A = A_o \frac{\sin\alpha}{\alpha} \text{ by } (1)$$

$$\Rightarrow R_o = A_o \frac{\sin\alpha}{\alpha} \frac{\sin N\beta}{\sin\beta}$$

Resultant Intensity $I = R_o^2$

$$\Rightarrow I = A_o^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin N\beta}{\sin\beta}\right)^2$$

~~Here First term~~ $\boxed{I = I_o \left(\frac{\sin\alpha}{\alpha}\right)^2 \left(\frac{\sin N\beta}{\sin\beta}\right)^2}$

Here first term $I_o \left(\frac{\sin\alpha}{\alpha}\right)^2$ is intensity due to single slit & second term $\left(\frac{\sin N\beta}{\sin\beta}\right)^2$ is due to combined Effect

Now i) Condition for maxima

$$\sin\beta = 0$$

or $\boxed{\beta = \pm n\pi}$

ii) Condition for Minima

$$N\beta = \pm m\pi$$

$$N\left(\frac{\pi}{\lambda}(a+b)\sin\theta\right) = \pm m\pi$$

$$\boxed{N(a+b)\sin\theta = \pm m\pi}$$

Diffraction grating:- An arrangement consist of large no. of close, Parallel straight and transparent Equidistant slits, each of Equal width 'a' seprated by an opaque region b.

The Spacing (a+b) between adjacent slit is called diffraction element or grating element.

$$\text{Grating element} = (a+b)$$

$$I = A^2 \frac{\sin^2 \alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2 \beta}$$

Resolving Power of an optical Instrument:

The ability or capability of an optical instrument to produce two separate images of two very close object is called resolving power.

OR

Ratio of wavelength of any spectral line to the smallest wavelength difference between very close line for which spectral line can just be resolved.

$$\text{Resolving Power} = \frac{1}{\Delta d} = \frac{2a}{\lambda}$$

(d- Distance between two object.
a- Numerical Aperture)

Dispersive Power:-

Rate of change of Angle of diffraction with wavelength of light used. $(d\theta/d\lambda)$

Rayleigh's Criterion:-

To obtain resolving power of an instrument Rayleigh suggested a criterion known as Rayleigh Criterion. According to Rayleigh criterion two images can be regarded as seprated if central maxima of one falls on the first minima of other.

Distance between the centre of patterns shall be equal to the radius of central disc. This is called Rayleigh limit of Resolution.

Resolving Power of a Plane transmission grating:-

$$\text{Resolving Power} = \frac{n\ell}{(a+b)} = nN$$

$$\left(N = \frac{\ell}{a+b}\right)$$

Resolving Power of Telescope:-

$$\boxed{r = \frac{1.22 f \lambda}{a}}$$

r - Radius of central bright image

a - Diameter of objective.

Resolving Power of Microscope:-

$$\boxed{d = \frac{\lambda_0}{2\mu \sin\alpha}}$$

λ_0 - wave length of light in vacuum

μ - Refractive index of Medium

$(\mu \sin\alpha)$ - Numerical Aperture of objective of Microscope.

Dispersive power	Resolving power
i) Defined as the rate of change of angle of diffraction with the wavelength of light used $(d\theta/d\lambda)$	i) Defined as the ratio of wavelength of any spectral line to the smallest wavelength difference between very close line for which the spectral line can just be resolved
ii) $\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$	ii) $\frac{\lambda}{d\lambda} = nN$
iii) It depends on $(a+b)$	iii) Independent of grating element $(a+b)$

Example 5.6 *Show that only first order spectra is possible if the width of grating element is less than twice the wavelength of the light.*

[GGSIPU, Dec. 2013 reappear (3 marks)]

Solution. Given $(a+b)<2\lambda$, suppose $(a+b)=(2\lambda-x)$; then grating formula

$$(a+b)\sin\theta = n\lambda$$

$$\theta = 90° \text{ for highest order}$$

$$(2\lambda - x) = n\lambda$$

$$n = \frac{2\lambda - x}{\lambda}$$

which is less than 2 or it is first order spectra.

Example 5.7 *A parallel beam of light is made incident on a plane transmission diffraction grating of 15000 lines per inch and angle of 2nd order diffraction is found to be 45°. Calculate the wavelength of light used.*

[GGSIPU, Dec. 2015 reappear (4.5 marks)]

Solution. Given : $N = 15000 \text{ lines/inch} = \frac{15000}{2.54} \text{ lines/cm}$,

$$n=2, \quad \theta = 45°, \quad \lambda = ?$$

We know the grating formula,

$$(a+b)\sin\theta = n\lambda$$

or
$$\lambda = \frac{(a+b)\sin\theta}{n} \quad ...(i)$$

$$(a+b) = \frac{1}{N} = \frac{2.54}{15000} \text{ cm} \quad ...(ii)$$

Putting Eq. (*ii*) in Eq. (*i*), we get

$$\lambda = \frac{2.54 \sin 45°}{15000 \times 2} = 5.987 \times 10^{-5} = 5987 \text{ Å}$$

Example 5.8 *A plane transmission grating has 15000 lines per inch. What is the highest order of the spectra which can be observed for wavelength 6000 Å? If opaque spaces are exactly two times the transparent spaces, which order of spectra will be absent ?*

[GGSIPU, Dec. 2015 reappear (3 marks)]

Solution. $N = 15000 \text{ lines/inch}$

$$(a+b) = \frac{2.54}{15000} \text{ cm}; \quad \lambda = 6000 \text{ Å} = 6.000 \times 10^{-5} \text{ cm}$$

We know the grating formula

$$(a+b)\sin\theta = n\lambda$$

For highest order, $\sin\theta = 1$

$$n = \frac{(a+b)}{\lambda} = \frac{2.54 \times 10^5}{15000 \times 6} = 2.8 \cong 3 \text{ (approximately)}$$

Hence the third order is highest order visible.

Problem 5.7 *A plane transmission grating having 6000 lines per cm used to obtain a spectrum of light from a sodium light in the second order. Find the angular separation between the two sodium lines ($\lambda_1 = 5890$ Å and $\lambda_2 = 5896$ Å).*

[GGSIPU, Dec. 2017 (5.5 marks)]

Solution. For diffraction grating,

$$(a+b) = \frac{1}{6000}\text{ cm} = \frac{1}{6000\times 100}\text{ m}, \quad \lambda_1 = 5.890\times 10^{-7}\text{ m}, \quad \lambda_2 = 5.896\times 10^{-7}\text{ m}$$

$(\theta_2 - \theta_1) =$ angular separation between two spectral lines = ?

$\because$ Condition for maxima,

$$(a+b)\sin\theta = n\lambda$$

$$(a+b)\sin\theta_1 = n\lambda_1$$

$$\sin\theta_1 = \frac{n\lambda_1}{(a+b)}$$

$$\Rightarrow \quad \theta_1 = \sin^{-1}\left[\frac{2\times 5.890\times 10^{-7}\times 6000\times 100}{1}\right] = 44°59'$$

and

$$(a+b)\sin\theta_2 = n\lambda_2$$

$$\sin\theta_2 = \frac{n\lambda_2}{(a+b)}$$

$$\Rightarrow \quad \theta_2 = \sin^{-1}\left[\frac{2\times 5.896\times 10^{-7}\times 6000\times 100}{1}\right] = 44°61'$$

Hence $\theta_2 - \theta_1 = 2'$.

Problem 6.14 *A 20 cm long tube containing sugar solution is placed between crossed Nicols and illuminated with light of wavelength 6×10^{-5} cm. If the specific rotation is 60°/dm/gm/cm^3 and optical rotation produced is 12°, determine the strength of the solution.*

[GGSIPU, Sept. 2011 (2 marks) ; Jan 2015 (3 marks)]

Solution. The specific rotation S of a solution is given by

$$[S]_T^{\lambda} = \frac{\theta}{l\times C}$$

Here, $\theta = 12°$, $l = 2.0$ dm and $S = 60/\text{dm/gm/cm}^3$

$$\therefore \quad C = \frac{12}{2.0\times 60} = 0.1\text{ gm/cc} = 10\%.$$

Problem 5.1 *Diffraction pattern of a single slit of width 0.5 cm is formed by a lens of focal length 40 cm. Calculate the distance between first dark and next bright fringe from the axis, $\lambda = 4890$ Å.*

[GGSIPU, Sept. 2012 (3 marks) ; Sept. 2013 reappear (4 marks)]

Solution. For Fraunhofer diffraction through narrow single slit, given

$$a = 0.5 \text{ cm} = 5\times10^{-3} \text{ m},$$

$$f = \text{focal length of lens} = 40 \text{ cm} = 0.4 \text{ m}$$

$$\lambda = 4890 \text{ Å} = 4.89\times10^{-7} \text{ m}$$

Distance between first minima and first secondary maxima $= x_2 - x_1$.

$\because$ Condition for minima is written as $a\sin\theta = n\lambda$

$$\Rightarrow \text{ for } n=1, \qquad \sin\theta = \frac{\lambda}{a} \text{ and } \sin\theta = \frac{x_1}{f}$$

$$\therefore \qquad x_1 = \frac{f\lambda}{a} = 3.912\times10^{-5} \text{ m.}$$

$\because$ Condition for secondary maxima is written as

$$a\sin\theta = (2n+1)\frac{\lambda}{2}$$

$$\Rightarrow \text{ for } n=1, \qquad \sin\theta = \frac{3\lambda}{2a} \text{ and } \sin\theta = \frac{x_2}{f}$$

Hence

$$x_2 = \frac{3f\lambda}{2a}$$

$$= \frac{3\times0.4\times4.89\times10^{-7}}{2\times5.0\times10^{-3}} = 5.868\times10^{-5} \text{ m}$$

$$\Rightarrow \qquad (x_2 - x_1) = 1.956\times10^{-5} \text{ m.}$$

Problem 6.5 *A plane polarised light is incident on a quartz plate cut parallel to the axis. Calculate the least thickness of the plate for which the o- and e-rays combine to form plane polarised light. Assume that $\mu_e = 1.5533$ and $\mu_o = 1.5442$ and $\lambda = 5.4\times10^{-5}$ cm* **[GGSIPU, Dec. 2015 (2 marks)]**

Solution. In this case the quartz plate must act as half wave plate. Thus if t be the required thickness then we have

$$(\mu_e - \mu_o)t = \frac{\lambda}{2}$$

or

$$t = \frac{\lambda}{2(\mu_e - \mu_o)}$$

Putting the given values, we get

$$t = \frac{5.4\times10^{-5}\text{ cm}}{2(1.5533-1.5442)}$$

$$= \frac{5.4\times10^{-5}\text{ cm}}{2\times0.0091} = 3\times10^{-3} \text{ cm.}$$

Example 5.5 *How many orders will be visible if the wavelength of an incident radiation is 5000 Å and number of lines on the grating is 2620 per inch ?* **[GGSIPU, Dec. 2013 reappear (2 marks)]**

Solution. Given $\lambda = 5000\ \text{Å} = 5.0 \times 10^{-7}$ cm,

$$N = 2620 \text{ LPI, then grating element } (a+b) = \frac{2.54}{2620} \text{ cm,}$$

We know grating formula $(a+b)\sin\theta = n\lambda$ (for highest order $\theta = 90°$), then $(a+b) = n\lambda$

or
$$n = \frac{(a+b)}{\lambda}$$

$$= \frac{2.54}{2620} \times \frac{1}{5.0 \times 10^{-5}} = 19.38 = 19$$

Example 5.10 *What is the least separation between wavelengths that can be resolved near 640 nm in the second order, using diffraction grating that is 5 cm wide and ruled with 32 lines per millimetre.*

[GGSIPU, Oct. 2013 (2 marks)]

Solution. Given $\lambda = 640$ nm, $n = 2$, $N = 32 \times 50 = 1600$, $d\lambda = ?$

We know resolving power of grating is given by

$$\frac{\lambda}{d\lambda} = nN$$

$$d\lambda = \frac{\lambda}{nN}$$

$$= \frac{640 \times 10^{-9}\ \text{m}}{2 \times 1600} = \frac{6400}{3200} \times 10^{-10} \text{ m}$$

$$= 2 \times 10^{-10} \text{ m} = 2\ \text{Å}$$

Problem 5.8 *Deduce the missing order for double slits Fraunhofer diffraction pattern, if the slit widths 0.16 mm and they are 0.8 mm apart.* **[GGSIPU, Sept. 2011 (2 marks)]**

Solution. Given that

$$a = 0.16 \text{ mm} \quad ; \quad b = 0.8 \text{ mm}$$

If a be the slit width and b the separation between slits ; the condition of missing order spectra is given by

$$\frac{a+b}{a} = \frac{n}{m}$$

$$\frac{0.16+0.8}{0.16} = \frac{n}{m}$$

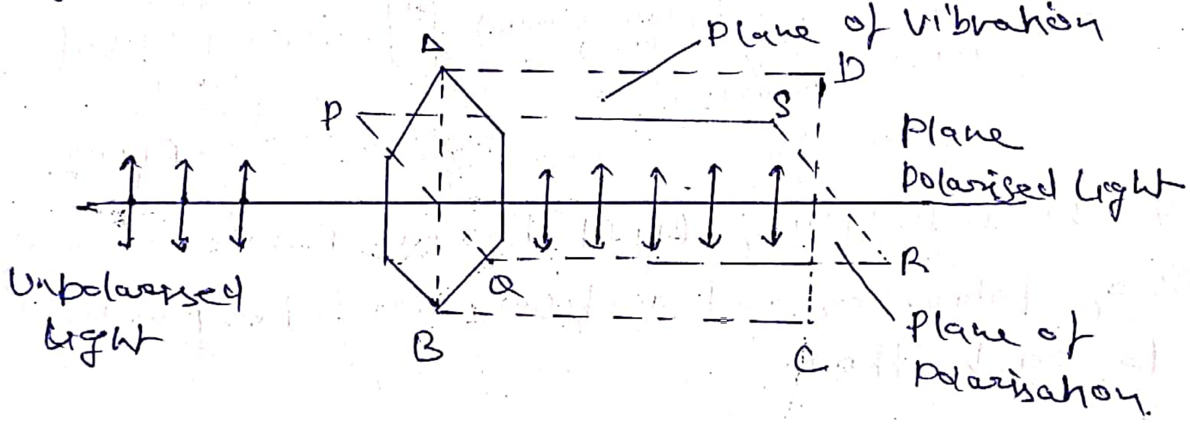
$$\frac{0.96}{0.16} = \frac{n}{m}$$

$$n = 6m = 6, 12, 18, \ldots \ (m = 1, 2, 3, \ldots)$$

Thus 6th, 12th, 18th, ... orders will be missing.

Polarization!- "The phenomenon due to which vibration of light are restricted in a particular plane is called polarisation of light."

When an ordinary light i.e Unpolarised light passed through a tourmaline crystal, out of all the vibrations which are symmetrical about the direction of propagation. only those pass through it which are parallel to crystallographic axis AB. Therefore direction of propagation are confined to a single plane.



Plane of Vibration!- The plane (ABCD) which contain vibration of plane polarised light is called plane of vibration.

Plane of Polarisation!- The plane (PQRS) perpendicular to the plane of vibration is called plane of polarisation.

Plane polarised light! It may be defined as the light in which vibration of light ~~are~~ are restricted to a particular plane.

Note!- Vibration of plane polarised light are perpendicular to the plane of polarisation.

Detection of Plane Polarised light:-

Polariser: - "Tourmaline Crystal or Nicol Prism used to produce plane polarised light called polariser.

Naked eye or the polariser can't make distinction between unpolarised light or plane polarised light.

Analyser: Crystal or Nicol Prism used to Analyse the Nature of light called Analyser.

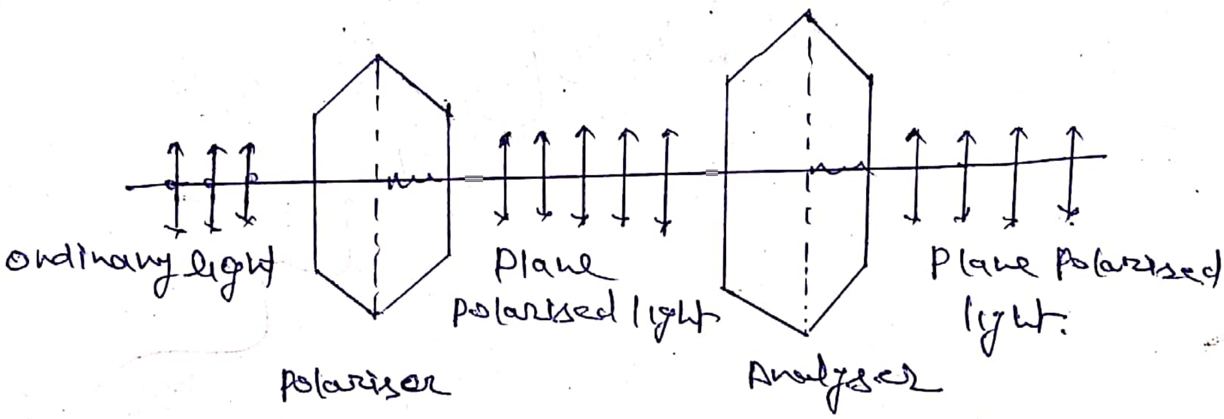


Fig. 1

* The intensity of light becomes Minimum when the axis of Polariser + Analyser are perpendicular to each other. (Fig 2)

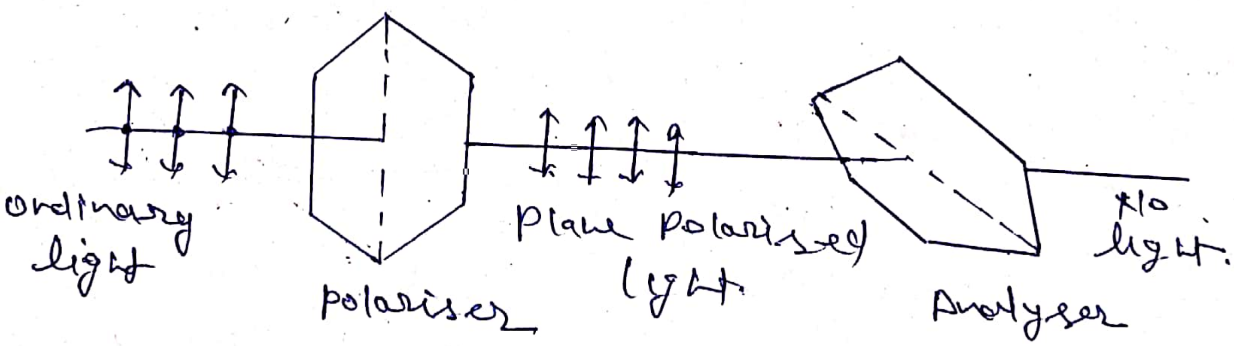


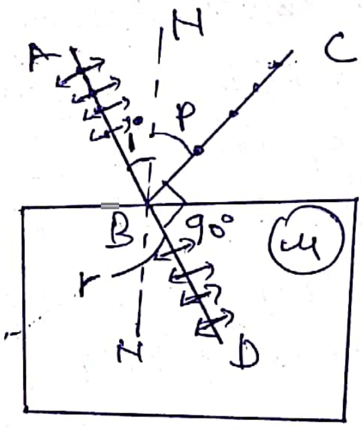
Fig. 2

<u>Brewster's Law</u>:→ It State that when light is incident at polarising angle at the interface of ~~reflecting~~ refracting, medium, the refractive index of Medium is equal to the tangent of the polarising angle.

If P be the Polarising angle and μ the refractive index of the refracting medium.

then according to Brewster's law.

$$\boxed{\mu = \tan P} \quad -①$$



When a light is incident at polarising angle P on a refracting medium of refractive index μ let r be the angle of refraction then according to Snell's law

$$\boxed{\mu = \frac{\sin P}{\sin r}} \quad -②$$

From ① & ②

$$\frac{\sin P}{\sin r} = \tan P$$

$$\frac{\sin P}{\sin r} = \frac{\sin P}{\cos P}$$

$$\Rightarrow \sin r = \cos P \quad \text{or} \quad \sin r = \sin(90 - P)$$

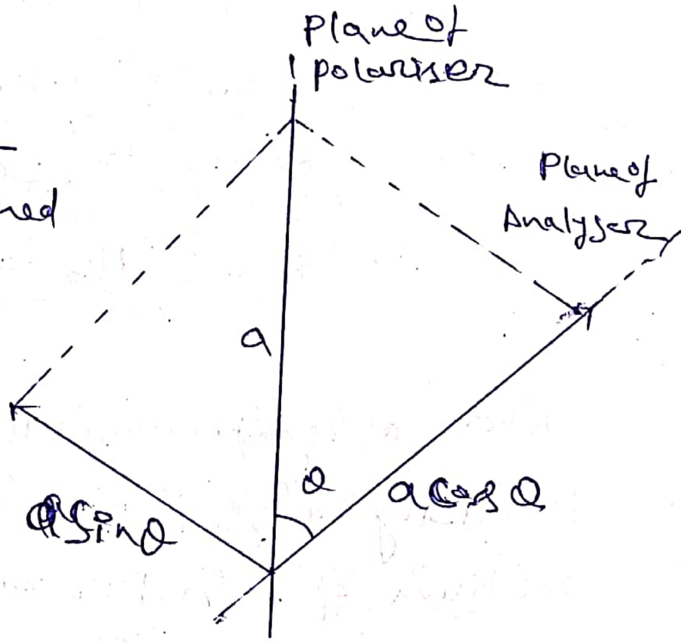
$$\Rightarrow r = 90^\circ - P \qquad \Rightarrow \boxed{r + P = 90^\circ}$$

Hence when a ray of light incident at polarising angle, the reflected ray is at right angle to the refracted ray.

Law of Malus:-

It state that when a completely Plane Polarised light beam is incident on an analyser, the intensity of the emergent light varies as the square of cosine of the angle between the plane of transmission of the analyser and polariser.

$$\boxed{I = a^2 \cos^2\theta}$$



Consider a plane of polariser and plane of analyser are inclined at an angle θ as shown in figure.

Further suppose that the plane of Polarised light of Intensity I_0 and amplitude a incident on polariser. Then

1) The component $a\cos\theta$ is along the plane of analyser.

ii) the component $a\sin\theta$ is along the perpendicular to the plane of analyser.

then

Intensity of light transmitted from analyser is given by -

$$I = a^2 \cos^2\theta$$

$a^2 = I_0$, intensity of incident plane Polarised light

therefore

$$\boxed{I = I_0 \cos^2\theta} \quad \text{or } I \propto \cos^2\theta$$

Condition:- when $\theta = 0$ or $180°$ $\cos\theta = \pm 1 \Rightarrow \boxed{I = I_0}$

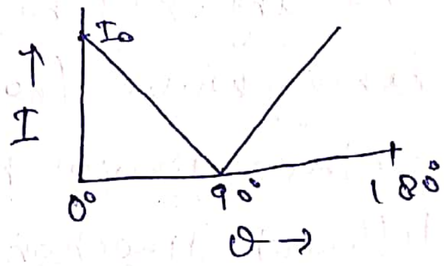
Hence when Polariser and Analyser are parallel, the intensity of light transmitted from ~~light~~ analyser is same as from polariser.

2) When $\theta = 90°$ so that $\cos\theta = 0$

$$\boxed{I = 0}$$

therefore when polariser and analyser are perpendicular the intensity of light transmitted from light is zero ie minimum

3) If we plot a graph between intensity of light and angle between polariser and Analyser it will be as -



4) In case light incident on analyser is unpolarised then $\boxed{I = \frac{1}{2} I_0}$

($\cos^2\theta = \frac{1}{2}$ $\cos^2\theta$ ie average value of $\cos^2\theta$)

Hence $I = \frac{1}{2} I_0$

(6) Nicol Prism:- "It is an optical device made from Calcite and frequently used for producing and analysis of Plane polarised light. It is based on the Phenomenon of double refraction."

Phenomenon of Double Refraction or Bire fringes:-

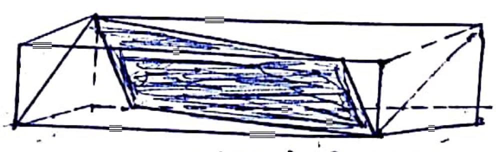
The Phenomenon of Splitting of unpolarised light into two polarised refracted ray is known as double refraction:

* When a Narrow beam of Unpolarised light be incident normally on a double refracting crystal such as calcite, it split into two refracted rays - one is 'ORDINARY RAY' or O-ray and other is 'EXTRORDINARY RAY' - E RAY.

Nicol Prism Principle

When unpolarised beam of light enter the calcite crystal it split into O-ray & E-ray.
In Nicol prism O-ray is Eliminated by total internal reflection. Hence E-ray only transmitted through the Prism.

Construction: A Calcite crystal cut into two half. The two half of crystal are properly polished and cemented in their original position. with a thin layer of cement named as Canada balsam.



Nicol Prism.

→ Refractive index of Canada Balsam (which is midway for E ray & O-ray)
$\mu = 1.55$

→ Refractive index for calcite of E-ray
$\mu_E = 1.49$

→ Refractive index for O-Ray $\mu_o = 1.66$

$\mu = \frac{1}{\sin C}$

$C = \sin^{-1}\left(\frac{1.55}{1.66}\right) \Rightarrow \boxed{C = 69^\circ}$

Critical Angle $C = 69^\circ$

~~Th~~ Huygen theory of Double Refraction:-

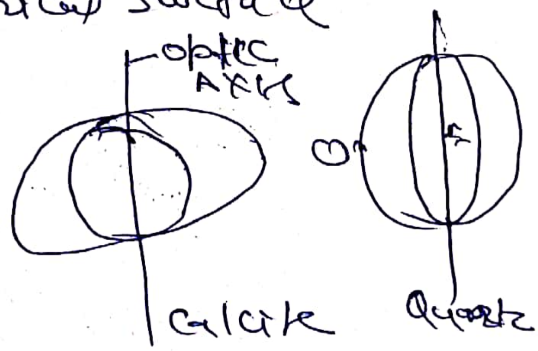
I) If two monochromatic ray is incident on a doubly ~~ref~~ refracting crystal it split into two wave front one for ordinary ray and other for extra ordinary ray.

II) ordinary ray has same velocity in all direction hence wave front in spheroidal.

III) Extra ordinary ray has different velocity in different direction hence its wave front is ellipsoidal

IV) ~~T~~ The uniaxial crystal has been classified as Negative and positive crystal. In Negative crystal like calcite extraordinary wave velocity (v_e) is greater than ordinary velocity (v_o).
In positive crystal $v_o > v_e$.

V) In Negative crystal like calcite ellipsoidal surface lies outside the spherical surface
In positive crystal ellipsoidal surface lies inside spherical surface.



VI) velocity of ordinary ray & Extra ordinary ray is same along optic AXIS.

VII) ~~Ellipsoidal~~ wave surface must be symmetrical about optic AXIS.

Optical Activity:- Certain crystals and solutions posses a Natural Ability to rotate the plane of polarization about the direction of its propagation. this process known as optical Activity

Right handed or dextro-rotatory:- Plane of Polarisation or Plane of vibrations Rotated in clockwise Direction.
Eg. Cinnabar, Cane Sugar etc.

~~Right~~ Left handed or Laevo rotatory:- Plane of Polarisation or plane of vibrations Rotated in anti clockwise direction.
Eg - Fruit Sugar

Specific Rotation:- Measure of optical Activity of a sample
Specific Rotation for a given wave length of light at a given temperature is defined conventionally as the Rotation produced by one decimeter long column of solution containing 1 gm of optically Active Material Per cc of solution.

$$(S)_\lambda = \frac{\theta}{l \times C} = \frac{\text{Rotation in Degree}}{\text{length in cc} \times \text{Conc in gm cc}}$$

Polarimeter:- A Polarimeter is an instrument used for determining the optical Rotation of Solution

→ When used for determining the optical Rotation of Sugar it is called Saccharimeter

Half Shade Polarimeter:- It consist of two Nicol Prism N1 & N2, N1 is polariser & N2 is analyser. Behind N1 is a half wave plate of quartz Q which cover one half of the view while other half is covered by Glass G.

Quater Wave Plate:- Plate of double refracting uniaxial crystal of calcite or quartz of suitable thickness whose refracting faces are cut parallel to the direction of optic Axis.

- If the thickness of the plate is t and the refractive index of ordinary and extraordinary rays are μ_o & μ_e respectively. then the path difference introduced between the two rays is given by $\lambda/4$

To produce a path Difference of $\lambda/4$ in Calcite

$$(\mu_o - \mu_e)t = \lambda/4$$

$$\boxed{t = \frac{\lambda}{4(\mu_o - \mu_e)}}$$

In Quartz $\boxed{t = \frac{\lambda}{4(\mu_e - \mu_o)}}$

Half wave Plate:- This plate is also made from double refracting uniaxial crystal of quartz or calcite with its refracting face cut parallel to the optic axis. The thickness of the plate is such that the ordinary and extraordinary rays have path Difference $= \lambda/2$

In Calcite $(\mu_o - \mu_e)t = \lambda/2$

$$\boxed{t = \frac{\lambda}{2(\mu_o - \mu_e)}}$$

In Quartz

$$\boxed{t = \frac{\lambda}{2(\mu_e - \mu_o)}}$$